MIMO SC-FDE Transmission Techniques with Channel Estimation and High-order Modulations

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Abstract — OFDM schemes have important limitations such as the high envelope fluctuations of the transmitted signals and its sensibility to carrier frequency errors. When these limitations bear critical we should consider SCFDE schemes (Single-Carrier Frequency Domain Equalization), which allow much higher power efficiency due to the lower envelope fluctuations. This can be further improved if the conventional linear FDE is replaced by an iterative FDE such as an IB-DFE (Iterative Block Decision Feedback Equalizer).

In order to obtain good performance results, accurate channel estimates must be available at the receiver, especially if MIMO (Multiple Input, Multiple Output) and high-order modulations are to be used. For this purpose, pilot symbols and/or training sequences are usually multiplexed with data symbols, which lead to spectral degradation. As an alternative, we can use implicit pilots.

In this paper, we consider SC-IB-FDE systems in MIMO $2 \times 2$ broadband with high order modulations and channel estimation.

1. INTRODUCTION

Block transmission techniques, with appropriate cyclic prefixes and employing FDE techniques (Frequency-Domain Equalization), have been shown to be suitable for high data rate transmission over severely time-dispersive channels \cite{1,2}. Two possible alternatives based on this principle are OFDM (Orthogonal Frequency Division Multiplexing) and Single Carrier (SC) modulation using FDE (or SC-FDE). Due to the lower envelope fluctuations of the transmitted signals (and, implicitly a lower PMEPR (Peak-to-Mean Envelope Power Ratio)), SC-FDE schemes are especially interesting for the uplink transmission (i.e., the transmission from the mobile terminal to the base station) \cite{1,2}, being considered for use in the upcoming LTE (Long Term Evolution) cellular system.

A promising IFDE (Iterative FDE) technique for SC-FDE, denoted IB-DFE (Iterative Block Decision Feedback Equalizer), was proposed in \cite{3}. This technique was later extended to diversity scenarios \cite{4} and layered space-time schemes \cite{5}. These IFDE receivers can be regarded as iterative DFE receivers with the feed-forward and the feedback operations implemented in the frequency domain. Since the feedback loop takes into account not just the hard decisions for each block but also the overall block reliability, error propagation is reduced. Consequently, IFDE techniques offer much better performance than non-iterative methods \cite{3–5}. Within these IFDE receivers the equalization and channel decoding procedures are performed separately (i.e., the feedback loop uses the equalizer outputs instead of the channel decoder outputs). However, it is known that higher performance gains can be achieved if these procedures are performed jointly. This can be done by employing turbo equalization schemes, where the equalization and decoding procedures are repeated in an iterative way \cite{6}, being essential in MIMO schemes with high order modulations. Although initially proposed for time-domain receivers, turbo equalizers also allow frequency-domain implementations \cite{7,8}.

In order for the above schemes to operate correctly, good channel estimates are required at the receiver. Typically, these channel estimates are obtained with the help of pilot/training symbols that are multiplexed with the data symbols, either in the time domain or in the frequency domain \cite{9–11}.

Overhead due to training symbols for channel estimation can be high, leading to decrease of system capacity, especially in fast-varying scenarios and/or high MIMO orders. A promising technique to overcome this problem is to use implicit training or implicit pilots, also called superimposed pilots, where the training block is added to the data block instead of being multiplexed with it \cite{12–16}. This means that we can increase significantly the density of pilots (to the maximum extent of one pilot per data symbol), with zero pilot overhead. Normally, periodic pilot sequences are added...
to data symbols in the time domain for single carrier systems [12, 16], or in the frequency domain for OFDM systems [13, 14].

In this paper, we consider the use of both multiplexed and implicit pilots for the SISO and MIMO settings with QPSK, 16QAM and 64QAM. For the implicit case, non-data-dependent pilots were used; i.e., the first approach mentioned above was used. We propose iterative receiver structures with joint channel estimation and detection. Unlike the iterative schemes of [16], our schemes do not employ the relatively complex Viterbi algorithm to jointly estimate channel and data. Instead, they incorporate iterative frequency domain equalization (either IB-DFE or turbo equalization) within the iterative channel estimation and detection/decoding framework.

The rest of this paper is organized as follows — the adopted system is introduced in Section 2 whilst Section 3 describes the proposed receiver structure. Furthermore, a set of performance results is presented in Section 4 and Section 5 contains the conclusions of this paper.

2. SYSTEM DESCRIPTION

We consider SC-FDE modulation. The $l$th transmitted block has the form

$$s_{n,tx}^l = \sum_{n=-N_G}^{N-1} s_{n,l}^{ntx} h_T(t - nT_s),$$

(1)

with $T_s$ denoting the symbol duration, $N_G$ denoting the number of samples at the cyclic prefix, $h_T(t)$ representing the adopted pulse shaping filter, and $s_{n,l}^{ntx}$ denotes the length-$N$ data block to be transmitted from the $ntx$ transmit antenna. After passing the signal to the frequency domain, the implicit pilots can be added. It is better to add them in the frequency domain, since most of the processing is done there.

The transmitted sequences are thus given by (capital $S$ being the signal in the frequency domain) [24]

$$X_{k,l}^{ntx} = S_{k,l}^{ntx} + S_{k,l}^{Pilot}$$

(2)

where, $S_{k,l}^{ntx}$ is the data symbol transmitted by the $k$th subcarrier (out of a total of $N$) of the $l$th FFT block and $S_{k,l}^{Pilot}$ is the corresponding implicit pilot. In Figure 1, we show a transmitter chain that incorporates for the SC-FDE scheme. The pilot symbols are added in the frequency domain, and the data is then passed through the IFFT for transmission (in practice, the resulting pilots could be added directly in the time domain, but Figure 2 makes things clearer).

Assuming only one user, the data bits are passed through a turbo coder, after which they are submitted to rate matching (taking into account the use of FFTs for faster processing, the antenna

![Figure 1: Transmitter scheme for the SC-FDE scheme.](image)

![Figure 2: Proposed frame structure for a MIMO-SC-FDE transmission with implicit pilots (P — pilot symbol, D — data symbol).](image)
multiplexer and block partitioning). All of the antennas will transmit a part of the message (if multiple users were to be employed, we could assign an antenna per user). The data bits are partitioned into blocks and the cyclic prefix is added to each block, so that the total size is a power of 2, for efficient use of the FFT.

We will consider the frame structure of Figure 2 for a SC-FDE system with $N$ carriers. According to this structure the pilot grid is generated using a spacing of $\Delta N_T$ symbols in the time domain (number of blocks) and $\Delta N_F$ symbols in the frequency domain. The minimum $\Delta N_F$ should be equal to the number of transmit antennas, so that there is no interference between pilot symbols on different antennas.

3. ITERATIVE RECEIVER

3.1. Receiver Structure

The transmission of pilot symbols superimposed on data will clearly result in interference between them. To reduce the mutual interference and achieve reliable channel estimation and data detection we propose a receiver capable of jointly performing these tasks through iterative processing. The structure of the proposed iterative receiver is shown in Figure 3. According to the figure, the signal, which is considered to be sampled and with the cyclic prefix removed, is converted to the frequency domain after an appropriate size-$N$ FFT operation. If the cyclic prefix is longer than the overall channel impulse response, the $nrx$ receive antenna is given as [17]:

$$R_{k,l,nrx} = \sum_{ntx=1}^{Ntx} \left( (S_{k,l,ntx} + S_{Pilot}^{k,l,ntx}) H_{k,l,ntx,nrx} + N_{k,l,nrx} \right)$$

with $H_{k,l,ntx,nrx}$ denoting the overall channel frequency response for the $k$th frequency of the $l$th time block between the $ntx$ transmit and $nrx$ receive antenna, and $N_{k,l,nrx}$ denoting the corresponding channel noise.

Before entering the equalization block, the pilot symbols are removed from the sequence resulting

$$(Y_{k,l,nrx})^{(q)} = R_{k,l,nrx} - \sum_{ntx=1}^{Ntx} S_{Pilot}^{k,l,ntx} (\hat{H}_{k,l,ntx,nrx})^{(q)}$$

where $(\hat{H}_{k,l,ntx,nrx})^{(q)}$ are the channel frequency response estimates and $q$ is the current iteration. Note that, in the case of a known channel without any pilots for estimation, $Y_{k,l,nrx} = R_{k,l,nrx}$.

The equalized samples are then simply computed as

$$\left(\hat{S}_{k,l,ntx}\right)^{(q)} = \left(\hat{H}_{k,l,ntx,nrx}\right)^{(q)*} \frac{Y_{k,l,ntx}^{(q)}}{|\hat{H}_{k,l,ntx,nrx}^{(q)}|^2}$$

where

$$Y_{k,l,ntx}^{(q)} = \sum_{nrx} (Y_{k,l,nrx})^{(q)} - \sum_{ntx1 \neq ntx} S_{Pilot}^{k,l,ntx1} (\hat{H}_{k,l,ntx1,nrx}^{(q)})$$

The sequences of the equalized samples are then passed through the IFFT, block grouping, demodulated and passed through the channel decoder. Each channel decoder has two outputs. One is the

![Figure 3: Iterative receiver structure.](image-url)
estimated information sequence and the other is the sequence of log-likelihood ratio (LLR) estimates of the code symbols. These LLRs are passed through the Decision Device which outputs either soft-decision or hard decision estimates of the code symbols. These estimates enter the Transmitted Signal Rebuilder which performs the same operations of the transmitter (coding, modulation). The reconstructed symbol sequence can then be used for improving the channel estimates, as will be explained next, for the subsequent iteration.

3.2. Channel Estimation Using Pilots

Let us first assume that \( S_{k,l} = 0 \), i.e., there is no data overlapping the training block, as in conventional schemes. In that case, the channel frequency response is, for a SISO (Single Input, Single Output — used solely for simplification purposes) scheme:

\[
\hat{H}_{k,l} = \frac{Y_{k,l}}{S_{k,l}^{TS}} = H_{k,l} + \frac{N_{k,l}}{S_{k,l}^{TS}} = H_{k,l} + \varepsilon_{k,l}^H
\]  

(7)

The channel estimation error \( \varepsilon_{k,l}^H \) is Gaussian-distributed, with zero-mean and

\[
E \left[ |\varepsilon_{k,l}^H| S_{k,l} \right] = E \left[ |N_{k,l}| \right] E \left[ \frac{1}{S_{k,l}^{TS}} \right]
\]  

(8)

3.2.1. Use of Implicit Pilots — General Case

Let us consider now the use of implicit pilots, i.e., \( S_k \neq 0 \) for the training blocks. In the following we will assume that

\[
E \left[ |S_{k,l}|^2 \right] = NE \left[ |s_n|^2 \right] = 2\sigma_D^2
\]  

(9)

and, for the frequencies that have pilots,

\[
E \left[ |S_{k,l}^{TS}|^2 \right] = NE \left[ |s_n^{TS}|^2 \right] = 2\sigma_P^2
\]  

(10)

Clearly, we will have interference between data symbols and pilots. This leads to performance degradation for two reasons:

• The data symbols produce interference on pilots, which might lead to inaccurate channel estimates. To reduce this effect, we should have

\[
\sigma_D^2 \ll \sigma_P^2
\]  

(11)

• The pilots produce interference on data symbols, which might lead to performance degradation (even if the channel estimation was perfect). To reduce this effect, we should have

\[
\sigma_P^2 \ll \sigma_D^2
\]  

(12)

Clearly, (11) and (12) are mutually exclusive. Moreover, the use of implicit pilots leads to increased envelope fluctuations on the transmitted signals [14]. To overcome these problems, we can employ pilots with relatively low power (i.e., \( \sigma_P^2 \ll \sigma_D^2 \)) and average the pilots over a large number of blocks so as to obtain accurate channel estimates. This is very effective since the data symbols have usually zero mean and different data blocks are uncorrelated. Naturally, there are limitations on the length of this averaging window, since the channel should be constant within it (not to mention the associated delays). Once we have an accurate channel estimate, we can detect the data symbols, eventually removing first the signal associated to the pilots.

Let us assume a frame with \( N_T \) time-domain blocks, each with \( N \) subcarriers. If the cyclic prefix of each FFT block has \( N_G = NT_g/T \) samples we will need \( N_G \) equally spaced frequency-domain pilots for the channel estimation. For pilot spacing in time and frequency \( \Delta N_T \) and \( \Delta N_F \), respectively, the total number of pilots in the frame is given by:

\[
N_{P,\text{Frame}}^F = \frac{N}{\Delta N_F} \cdot \frac{N_T}{\Delta N_T}
\]  

(13)
This means that we have a pilot multiplicity or redundancy of

\[
N_R = \frac{N_F^{\text{Frame}}}{N_G} = \frac{N}{N_G \Delta N_F} \cdot \frac{N_T}{\Delta N_T}
\]  

(14)

Therefore, the SNR associated to the channel estimation procedure is

\[
\text{SNR}_{\text{est}} = \frac{N_R \sigma_T^2}{\sigma_N^2 + \sigma_D^2} = \frac{N_R \sigma_T^2}{\sigma_D^2} \cdot \frac{\text{SNR}_{\text{data}}}{1 + \text{SNR}_{\text{data}}}
\]

(15)

where \( \sigma_N^2 = \frac{1}{2} E[|N_{k,l}|^2] \) and the SNR associated to data symbols is given by \( \text{SNR}_{\text{data}} = \sigma_D^2/\sigma_N^2 \). For moderate and high SNR values,

\[
\text{SNR}_{\text{est}} \approx N_R \frac{\sigma_T^2}{\sigma_D^2}
\]

(16)

i.e., we have an irreducible noise floor of \( N_R \sigma_T^2/\sigma_D^2 \). To avoid significant performance degradation due to channel estimation errors, \( \text{SNR}_{\text{est}} \) should be much higher than \( \text{SNR}_{\text{data}} \). This could be achieved with \( \sigma_T^2 \ll \sigma_D^2 \), provided that \( N_R \gg 1 \).

3.2.2. MIMO Estimation Algorithm with Implicit Pilots

To obtain the frequency channel response estimates the receiver applies the following steps in each iteration:

a. Data symbols estimates are removed from the pilots. The resulting sequence becomes

\[
\left( \hat{R}_{k,l,ntx} \right)^{(q)} = R_{k,l,ntx} - \sum_{ntx=1}^{Ntx} \left( \left( \hat{S}_{k,l,ntx} \right)^{(q-1)} \left( \hat{H}_{k,l,ntx,ntx} \right)^{(q-1)} \right)
\]

(17)

where \( \left( \hat{S}_{k,l,ntx} \right)^{(q-1)} \) and \( \left( \hat{H}_{k,l,ntx,ntx} \right)^{(q-1)} \) are the data and channel response estimates of the previous iteration. This step can only be applied after the first iteration. In the first iteration we set \( \left( \hat{R}_{k,l,ntx} \right)^{(1)} = R_{k,l,ntx} \).

b. The channel frequency response estimates is computed using a moving average with size \( W \), whilst at the same time removing the pilots, as follows (data is considered to be zero mean):

\[
\left( \hat{H}_{k,l,ntx,ntx} \right)^{(q)} = \frac{1}{W} \sum_{l'=-[W/2]}^{l+[W/2]} \left( \hat{R}_{k,l',ntx} \right)^{(q-1)} \frac{\left( \hat{S}_{k,l',ntx} \right)^{(q-1)}}{S_{\text{Pilot}}^{\text{Frame},ntx}}
\]

(18)

c. After the first iteration the data estimates can also be used as pilots for channel estimation refinement. This is especially useful if the spacing of pilot symbols in the time domain is \( \Delta N_T > 1 \). The respective channel estimates are computed as

\[
\left( \hat{H}_{k,l,ntx,ntx} \right)^{(q)} = \frac{\left( Y_{k,l,ntx} \right)^{(q-1)} \left( \hat{S}_{k,l,ntx} \right)^{(q-1)^*}}{\left( \left( \hat{S}_{k,l,ntx} \right)^{(q-1)} \right)^2}
\]

(19)

d. These channel estimates are enhanced by ensuring that the corresponding impulse response has a duration \( N_G \). This is accomplished by computing the time domain impulse response of (18) and (19) through \( \{\hat{h}_{i,l}\}^{(q)}; i = 0, 1, \ldots, N-1 \} = \text{IDFT}\{\hat{H}_{k,l}\}^{(q)}; k = 0, 1, \ldots, N-1 \} \) (zeros can be used for the missing carriers if \( \Delta N_F > 1 \) in order to perform a “FFT-interpolation”), followed by the truncation of this sequence according to \( \{\hat{h}_{i,l}\}^{(q)} = w_i \hat{h}_{i,l}^{(q)}; i = 0, 1, \ldots, N-1 \} \) with \( w_i = 1 \) if the \( i \)th time domain sample is inside the cyclic prefix duration and \( w_i = 0 \) otherwise. The final frequency response estimates are then simply computed using \( \{\hat{H}_{k,l}\}^{(q)}; k = 0, 1, \ldots, N-1 \} = \text{DFT}\{\hat{h}_{i,l}\}^{(q)}; i = 0, 1, \ldots, N-1 \} \ast \Delta N_F \).
4. NUMERICAL RESULTS

The number of carriers employed was $N = 256$, each carrying a QPSK/16QAM/64QAM data symbol. Each information stream was encoded with a variable block size per antenna, yielding a deterministic number of 256-bit blocks after the FFT conversion. For the implicit case, the overall block size was of 2880 bits, whereas for the multiplexed pilots case, it was 720 bits per antenna — this way we had a fixed number of blocks per antenna for the multiplexed pilots case, and the same overall amount of bits for the implicit case, in order to avoid coding gains. The multiplexed pilots case used an extra block dedicated for channel estimation. The channel impulse response employed is characterized an exponential PDP (Power Delay Profile) with 32 symbol-spaced taps and normalized delay spread $\sum_{i=0}^{31} 10 \log_{10} (e^{(t_i - t_0)/\tau})$. A symbol duration of $T_s = 260$ ns was used.

The channel encoders were rate-1/2 turbo codes based on two identical recursive convolutional codes with two constituent codes characterized by $G(D) = (1 + D^2 + D^3)/(1 + D + D^3)$. A random interleaver was used within the turbo encoders.

Most of the BER (Bit Error Rate) results presented next will be shown as a function of $E_s/N_0$, where $E_s$ is the average signal energy and $N_0$ is the single sided noise power spectral density. For channel estimation purposes, the moving average window size used was $W = 9$, considering different values of power ratio $\beta P$. The figures combine the use of perfect channel estimation, with estimation using multiplexed pilots (using $\Delta N_F = ntx$ and $\Delta N_T = N_T$) and estimation using implicit pilots (using $\Delta N_F = ntx$ and $\Delta N_T = 1$).

The power of the pilots is taken to be the same as the mean symbols’ power throughout the block for the explicit case, and have a value of $-6$ dB for the implicit case. A fixed sliding window size of 45 was chosen.

In Figures 4–6, all the main performance results can be observed. Note that for the 16QAM and 64QAM, there are different orders of bit performance, and thus results were split into MSB — Most Significant Bit (highest protection); ISB — Intermediate Significance Bit (intermediate protection, only for 64QAM that modulates 3 bits) and LSB — Least Significant Bit (lowest protection).

In Figure 4, explicit channel estimation yields results similar to perfect estimation, and implicit estimation has a drawback of 2–3 dB. Note that the performance requires higher power for higher modulations, and that the offsets of using estimations also increase.

Comparing Figure 4 with Figures 6, we note that the MIMO $2 \times 2$ yields better results than SISO, due to high diversity and good equalization receiver. Between Figures 5–6 with a higher speed, the results require marginally more power, easily explained by the extrapolation error caused between pilot-exclusive blocks.

As regards for the implicit pilot estimation case, it can be seen throughout that it performs reasonably for modulations up to 16QAM (or 64QAM for SISO with “low” speed $v = 100$ km/h), with all its natural advantages of using the full bandwidth for data transmission — and a small amount of power for implicit pilot estimation.

![Figure 4: BER values for SC-FDE using IBDFE, SISO, QPSK/16QAM/64QAM, $v = 100$ km/h.](image-url)
5. CONCLUSIONS

In this paper, we have studied the use of SISO and MIMO $2 \times 2$ QPSK/16QAM/64QAM turbo-coding in a SC-FDE system employing both multiplexed and implicit pilots with the aim of supporting multicast and broadcast transmissions. To deal with the problem of the mutual interference between pilots and data symbols, which can severely affect the performance of QAM modulations, we proposed the use of an iterative receiver capable of accomplishing joint channel estimation and data detection. It was verified through simulations that, using channel estimation both with multiplexed pilots (at the cost of pilot bandwidth) and with implicit pilots (at the cost of around $-6$ dB more power), good performance results can be obtained.

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