Market-Based Allocation of Airport Slots: The PAUSE Auction Mechanism and Extensions

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ABSTRACT

During the recent past, passenger air transport has been recovering from its significant retraction during the Covid-19 pandemic. If the recent significant drop in air traffic due to the pandemic acted as an external mitigating factor to airport traffic congestion in several major airports around the world, with the post-pandemic air traffic recovery airport capacity is likely to, once again, fall short of demand and not keep pace with the growth in air traffic. For this reason, close to two hundred major airports worldwide, most of them in Europe, face capacity constraints. Eurocontrol predicts Europe's capacity shortage in 2050 at 500,000 flights/year given the baseline scenario, which could rise to 2.7 million based on an optimistic scenario.

The allocation of airport slots in Europe and elsewhere is still ruled by administrative processes, based on IATA's Guidelines, which follow historical precedence and time adjustments of historical slots. Market mechanisms in slot allocation, as an alternative to administrative processes, are controversial and still rarely used. Several authors have highlighted the inefficiency of the current airport slot administrative allocation system, based on those guidelines. Some have suggested improvements within this administrative system, others have suggested new mechanisms altogether, such as congestion pricing mechanisms and other market mechanisms involving auction procedures. Among the various auction mechanisms, scoring auctions and the Progressive Adaptive User Selection Environment (PAUSE) methodology have been suggested.

In this paper, and following our previous work, we explore and extend the application of the PAUSE auction mechanism with bidding based on a score function for the auctioneer, which includes another variable in addition to total revenue, where this variable represents the quality of the service provided. We suggest the application of this auction mechanism, in a gradual fashion, to the three international airports operating in Portugal that are level 3 all year round. The different airlines using these airports would still follow the current IATA guidelines during their use of other airports, including the slot exchange protocols. We suggest that some of the PAUSE auction mechanism's desirable properties, such as computability, transparency, absence of envy, and the mitigation of the "price-jump problem", "threshold problem", "exposure problem", and "winner's curse problem", still hold.

I. Introduction

"Auctions are used when the seller does not know the bidders' willingness to pay for idiosyncratic items for which there is no well-functioning market."[1]

"There is now a combinatorial auction procedure that can handle slot allocation problems of any size. We should no longer put off introducing auctions for airport slot allocation" [2]

DURING THE PAST SEVERAL MONTHS, passenger air transport has been recovering from its significant retraction during the two-year Covid-19 pandemic. From a total of 4,543 million passengers worldwide in 2019, the actual number in 2020 was 1,807 million, i.e., around 40% of the number in 2019 and around 38% of the initial forecast of 4,723 million for 2020. In 2021, the number increased to 2,277 million, and is expected to have reached 3,432 million by the end of 2022, i.e., around 73% of the pre-pandemic forecast for 2020 and around 76% of the total actual number in 2019.

If the recent significant drop in air traffic due to the Covid-19 pandemic acted as an external mitigating factor to airport traffic congestion in several major airports around the world, with the post-pandemic air traffic recovery, it is likely that airport capacity will, once again, fall short of demand and not keep pace with the growth in air traffic. In Europe, where more than 46% of level 3 airports[3] are located (but only 32% of all IATA international airports), Eurocontrol still foresees airport capacity constraints that will impact almost 0.5 million flights/ year by 2050 given the most likely scenario (up to 2.7 million based on the high scenario[4]), even though this level of unmet demand was previously forecast for 2035 before the pandemic crisis (Eurocontrol, 2022).

Given such a capacity constraint scenario, and as argued in Cardadeiro & Gata (2021) an efficient allocation of scarce airport capacity is critical for air traffic growth, as well as for the overall dynamic efficiency of air transport.

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[1] See McAfee, R. P. & McMillan, J. (1996), p. 173.

[2] See Steinberg, R. (2021), p. 77. See also https://cepr.org/voxeu/columns/it-time-auction-slots-congested -airports.

[3] Airports throughout the world are categorized as Level 1 (Non-Coordinated Airport), Level 2 (Schedules Facilitated Airport) or Level 3 (Coordinated Airport). Additionally, in some states or areas, such as in the European Union, legally binding regulations must be observed. Some airports are classified as level 3 for specific days of the week only, or for a particular season, and not for the whole year. For example, Faro airport is level 3 only during the summer season. However, Lisbon, Porto and Funchal airports are level 3 for the whole year. 'Coordinated airports' are airports subject to coordination as a means of managing airport capacity, where coordination involves the allocation of constrained or limited airport capacity to airlines and other aircraft operators to ensure a viable airport and air transport operation. It is also presented as a process employed to maximize the efficient use of airport infrastructure.

[4] The flight forecast for Europe's base scenario predicts 16 million flights in 2050, a 1.2% average annual growth from 2019, a level of activity that was previously (pre-covid) brought forward to 2040. The high scenario foresees an 1.8% average annual growth.

Indeed, more efficient management of existing capacity could alleviate pressure on airport capacity for around 80 level 3 airports in Europe. Furthermore, a more efficient allocation of airport capacity improves the impact of the aviation industry on the economy and its overall contribution to reducing CO_2 emissions and other types of harmful emissions.

However, and as is well known, the allocation of airport slots in Europe and elsewhere is still ruled by administrative processes, based on IATA's WASG[5], which follow historical precedence (called "grandfather rights") and time adjustments of historical slots. Market mechanisms in slot allocation, as an alternative to administrative processes, are still rarely used. In particular, the use of auctions for slot allocation has only been implemented in some local routes in China (Sheng et al., 2015), and apparently this underemployment of auction mechanisms has been due to the reluctance of coordination authorities to face the risks that have been pointed out regarding the long-term route planning of airlines, the usage costs related to excess slots, origin-destination pairing, and the impact and competition distortions of air transport fares (Button, 2008; Sentance, 2003).

Several authors have highlighted the inefficiency of the current airport slot administrative allocation system, based on IATA's WASG. Many have suggested improvements in this administrative system (Katsigiannis et al. 2021), as well as congestion pricing mechanisms (Noto, 2020) and other market mechanisms involving auction procedures (Ball et al., 2018). Among the various auction mechanisms, scoring auctions and the Progressive Adaptive User Selection Environment (PAUSE) methodology have been suggested in the literature (Steinberg, 2021).

In this paper we explore the application of a PAUSE auction mechanism with bidding based on a score function that includes other variables in addition to the auctioneer's total revenue. We suggest a preliminary application of this mechanism to the three international airports operating in Portugal[6] namely Lisbon, Oporto and Funchal that are classified as level 3 year-round, focusing on a more efficient slot allocation, whether they are departure or arrival slots. The different airlines using these airports would still follow the current IATA guidelines in their use of other airports, including the slot exchange protocols as they are provided.

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[5] Worldwide Airport Slot Guidelines.

[6] According to the Portuguese Authorities, passenger traffic in Portugal plummeted 70% in 2020 compared to 2019 and recovered slightly to 60% in 2021. During the first semester 2022, it still lagged behind the 2019 correspondent figures by 13%. By the end of the year, it had almost reached the 2019 level (-3%). See https://www.nav.pt/espaco-aereo/dados-de-trafego.

II. Slot Allocation Mechanisms

2.1 The Current Administrative Slot Allocation

As is well known, the allocation of "slots" in most airports in Europe, including in Portugal, and other jurisdictions, such as the USA (with some exceptions), is governed by administrative processes based on IATA's WASG, which follow historical precedence and time adjustments of historical slots, and not market mechanisms.

Slot allocation is used in the most congested (or level 3) airports to allocate and manage a limited airport capacity, with the aim of maximizing the "efficiency of an airport". In Council Regulation (EEC) No. 95/93, from January 18, on the common rules applicable to the allocation of slots in airports within the European Community (today's European Union/EU), a "slot" is defined as "the estimated time of arrival or departure available or assigned to an aircraft movement on a specific date at a coordinated airport" (see its Art. 2(a))[7]. A coordinator is appointed to facilitate air carrier operations at that airport.

Airport coordinators assigned to a level 3 airport manage scarce airport capacity. These airport coordinators are functionally and financially independent of any interested parties, including governments, airlines and airports, applying the rules set out in WASG. They are responsible for affecting airport slots and supervising their use in a neutral, transparent and non-discriminatory manner. The temporal duration of each "slot" is based on block placement (arrival) and block removal (departure) times. Actual landing and take-off times may vary due to operational factors.

These rules are closely followed by law in the US, the EU, and many other jurisdictions, and are supported by two principles: the principle of "grandfather rights" (which establishes 'historical precedence'), and the "use-it-or-lose-it" principle. The first establishes that "slots" are assigned to flight operators that have already had these same "slots" assigned in the previous corresponding "IATA season", following the historical schedule at that airport. The second introduces a restriction to the 'grandfather rights' rule, as it states that this rule only applies to "slots" (or a series of "slots") that have been effectively used by the flight operator in at least 80% of those affected time intervals[8]. It is for this reason that the "use-it-or-lose-it" rule is also known as the "80/20 rule".

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[7] In the US, the term "slot" means "a reservation for an instrument flight take-off or landing by an air carrier of an aircraft in air transport". In the EU, the definition of a "slot" is more demanding: it includes not only an authorization to access a "slot" for take-off or landing on specific dates and times, but also the associated airport infrastructure, which is left out of the definition used in the US, where such resources must be obtained by airlines through other means.

[8] Due to the Covid-19 pandemic, and recently to the invasion of Ukraine by the Russian Federation, this rule has been temporarily relieved, in the EU by Regulation (EU) No. 2020/459 of March 30, Regulation (EU) No. 2021/250 of February 16, Regulation (EU) No. 2021/1889 of July 23, Regulation (EU) No. 2022/255 of December 15, 2021, and Regulation (EU) No. 2022/2038, of October 30.

WASG are a set of standards and best practices developed by IATA member airlines together with the community of airport coordinators and facilitators. These guidelines are a comprehensive set of procedures for the allocation and management of airport capacity at various airports worldwide. The main users of these guidelines are airlines and airport coordinators and facilitators. IATA maintains the WASG under the supervision of the Joint Slot Advisory Group (JSAG). All changes are agreed upon by the JSAG before being presented to the IATA Slot Conference Heads of Delegation for majority approval. This ensures that no new or amended standards or best practices can be introduced into WASG unilaterally by any airline, coordinator or even IATA.

WASG are recognized by the global air transport community as the industry standard methodology for allocating slots at level 3 airports and managing planned operations at level 2 airports. If a state or a government intends to develop and implement its own rules or procedures, the principles of these guidelines should be adopted. IATA encourages the use of these guidelines as a basis for these regulations.

IATA claims that WASG provide a consistent, transparent, and fair method for allocating and managing airport capacity, given that air transport is global in nature and requires harmonized slot allocation rules, both at the origin and destination airports of each route, to maximize efficient use of resources by an airline.

The IATA Slots Conference (SC) is the forum for the coordination of planned operations at level 2 and level 3 airports, held twice a year in the summer and winter seasons (of the northern hemisphere). The June SC the covers the following winter season, and the November SC covers the following summer season. The IATA SC is only convened to allocate and manage slots at level 3 airports and discuss schedule adjustments at level 2 airports from a competitive standpoint. Changes beyond the scope of the SC are not permitted. SC dates are communicated to industry by IATA Management using the Calendar of Coordination. Other essential dates in the coordination process can also be consulted on this calendar. Delegates attending the SC must be accredited representatives of their airline or coordinating organization. The SC is essential to help ensure that airlines are successful in terms of operating commercially and the management of each airport's capacity by coordinators and facilitators. IATA strives to ensure that the information necessary to participate in the SC is available to all airlines, coordinators, and facilitators. IATA maintains a directory of companies participating in the SC and the individuals designated by their companies as Heads of Delegation. This directory lists the names and contact details of coordinators and facilitators and those who are authorized to transfer, exchange, change and delete or request new slots on behalf of an airline participating in the SC.

Different airlines may still transfer or exchange slots with each other and may use them as part of a shared operation, subject to these guidelines and other existing regulations. This possibility creates a semblance of a "secondary market": exchanges and transfers are not monetized. At level 3 airports, airline-

-allocated slots may be exchanged between airlines, but only on a one-for- one basis.

Finally, all "slots" that are not affected by these rules, newly created "slots" that for some reason become available, join a "pool" of "slots" and are allocated to airlines by the coordinator based on a set of hierarchical rules that include 50% of "slots" for new entrants, consideration of year round operations and various other factors such as the type of customer or market served, connectivity, competitive forum, etc. These multi-purpose (potentially conflicting) factors provide additional degrees of freedom to consider public interest aspects related to the allocation of slots, other than the reservation of certain slots for regional services or services with public service obligations.

However, and as referred to by NERA back in 2004[9]:

«Despite this excess demand [in level 3 airports], the effect of the current EU slot regulation (95/93) is that existing users of these airports enjoy grandfather rights in relation to their current slot holdings. While the regulation provides for 50 per cent of any slots not subject to grandfather rights to be allocated to new entrants, and the remainder allocated according to administrative criteria, there are usually very few slots available from this "pool". It is therefore difficult for airlines to obtain slots in order to introduce new or more frequent services. The fact that existing airport charges fail to reflect the scarcity value of slots means that they may be allocated to services that are barely profitable at the current level of airport charges. Airlines that might be able to use these slots more efficiently (for example, because their services would carry more passengers or generate more profits) and would therefore be willing to pay considerably more than the current level of charges, may nevertheless be unable to get hold of any slots. In addition, the fact that airlines only pay charges based on their actual use of slots means that they have poor incentives to use slots efficiently. Some slots therefore remain unused, even at congested airports – either because they are returned late to airport coordinators (and cannot be reallocated) or because airlines simply fail to use their full allocation of slots»[10]

More recently, A. Ockenfels et al. (CEPR, Jan. 2021) state that because airport time slots are currently awarded by historic use, with only a small number reserved for new entrants, competition is being hampered, together with an inefficient slot utilization, contributing to congestion at airports.

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[9] See NERA (2004).

[10] Concerning the role of market mechanisms: *Market mechanisms have the potential to address these inefficiencies by confronting airlines with the cost of occupying limited capacity. If market mechanisms were introduced at congested EU airports, we [NERA] believe that higher passenger volumes would use existing airport facilities.*

2.2 Airport Congestion Pricing

Congestion pricing requires that slot prices be defined a priori, as many prices as necessary (peak load prices, off-peak load prices, and super-peak load prices, etc.). This definition can be quite challenging. However, in the case of slot auctions, the number of slots to be auctioned needs to be specified a priori, which is a simpler task.

As reported in Cardadeiro & Gata (2022), several authors have suggested the use of congestion pricing, but its merits compared to other slot allocation approaches are not consensual as they depend on several assumptions on the demand for slots, airline pricing policy, airline market structure/concentration/ differentiation, airport integration into the airport's network, information available for slot price setting and other modeling and market details. Setting appropriate congestion prices in a dynamic setting is a highly complex problem, even if this allocation mechanism has the advantage of allowing increased carrier scheduling flexibility and reducing the incentives for airlines to hoard airport slots. Additionally, the use of prices to manage demand and accommodate airport capacity is usually constrained by economic regulation of operations within the aviation industry, as at most level 3 airports, airport tariffs, including slot prices, are regulated to control the monopoly power of airports, directly or indirectly capping slot prices, either using an incentive or a cost-plus regulation model. However, some argue that the use of congestion pricing, if appropriately designed, might be compatible with the existing regulatory practice. The debate on the benefits of congestion pricing is enriched by those arguments that suggest combining congestion pricing with other mechanisms to improve efficiency, as each of the different alternatives alone is not sufficient.

2.3 The Use of Pause to Auction Airport Slots

Auctions have been proposed as an airport slot allocation mechanism for more than four decades[11] and, despite the complexity of slot allocation among airlines and some

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[11] As mentioned in Cardadeiro & Gata (2021), in several instances auction mechanisms have been used to allocate airport slots. In 2016, the Civil Aviation Administration of China initiated a trial slot auction scheme for domestic flights at Guangzhou Baiyun and Shanghai Pudong Airports. Auction winners retained the slots for three years. In November 2011 when US Airways and Delta Air Lines were required by the US Department of Treasury to divest a total of 48 slots, or 24 slot pairs, at the Ronald Reagan Washington National Airport and NY LaGuardia Airport, the US FAA employed simple auction mechanisms to execute these transactions, for which the buyers and sellers were airlines. These were secondary market transactions where the government played a forcing role – see M. Ball et al. (2018). In December 2021, the European Commission approved a €2.55 billion Portuguese restructuring aid in favor of the TAP Group, including TAP Air Portugal, under the EC's Guidelines on State aid for the rescue and restructuring of companies in difficulty, subject to several conditions that included the transfer of up to 18 slots per day at the Lisbon airport held by TAP, to a competing carrier. On 25 February 2022, the Monitoring Trustee, appointed by the TAP Group, announced a call for proposals (sealed-bid auction) for interested carriers. The EC, assisted by the Monitoring Trustee, assessed the proposals received against the eligibility, evaluation and ranking criteria set out in the Commission decision of 21 December 2021, and ranked the proposals based on the seat capacity that carriers would offer using the slots made available by TAP. Based on this assessment, the EC ranked easyJet's proposal first.

drawbacks, the theory of auctions has evolved significantly (Milgrom, 2021) providing several tools that could improve the efficiency of how airports use and manage limited capacity (Rassenti et al., 1982; Button, 2008; Basso et al., 2010; Sheng et al., 2015; Ball et al.,2018). Cardadeiro & Gata (2021) suggest a combinatorial scoring auction approach, with multiple phases, as an attempt to mitigate some of the criticism regarding previous slot auction proposals, namely the clock-proxy one, and argue, along with other authors (e.g., Ockenfels et al., 2021) that the transitory Covid-driven air traffic reduction should be considered a window of opportunity to implement auctions for slot allocation in congested airports. Steinberg (2021) also supports the use of auctions for this purpose, but further suggests the use of PAUSE as the appropriate auction mechanism, as described by Land et al. (2006).

Prior to our contribution to the auction mechanism design using PAUSE, we shall start by stating that we consider the adoption of auctions for slot allocation as an incremental, or progressive, approach to the existing allocation mechanisms, in order not only to mitigate the eventual risks associated with airline operational disruption, but also to test and progressively improve the slot auction design. In this respect, we are aligned with Ball et al. (2018) regarding auctioning only 5% to 10% of airport slots, on an annual basis, revising the concept of the existing slot rights to a "limited time lease right" and many other aspects designed to promote a smooth transition from the existing allocation method to an auction-based method.

This means that auctioning 5% to 10% of the slots each year will not conflict with all the other existing slot exchange practices and secondary market mechanisms, namely the IATA conferences held twice a year. Furthermore, this means that we consider the use of auctions as a tool to improve allocation efficiency, from the existing standpoint, and not the first-best allocation mechanism, which would not be possible to implement.

That being said, we would like to consider the PAUSE auction mechanism, a discrete-time combinatorial auction (more specifically, a discrete-time combinatorial, simultaneous, progressive, bottom-up auction[12]) containing several stages. Each stage can contain several bidding rounds ("rounds"). The PAUSE mechanism was proposed by Kelly & Steinberg (2000) to be applied in the allocation by auction (with at least two stages) of universal telecommunication services in the various geographic areas of the USA (i.e., a "carrier of last resort", or COLR). The auctioneer can be a suitable programmed algorithm.

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[12] The auction is called *combinatorial (or composite)* because it allows bidding to take place on bundles of lots and not just on individual lots. It is *simultaneous* because in each bidding round the bidders bid simultaneously, that is, each bidder bids on the lots made available by the auctioneer without observing the bidding of the other bidders. It is *progressive* because at each stage, each bidder only has access to the next bidding round if he bid more in that round than he bid for each lot or package of lots in the previous round ("improvement margin requirement"). As it is a progressive auction, it is also ascending, as the prices of the lots increase throughout the auction. However, it is also progressive in the sense that the maximum size of packages (of lots), bid on during each stage, also increases as each stage is completed, until the conclusion of the auction.

In terms of a COLR auction, firms bid for subsidies in geographic areas to guarantee the implementation of a Universal Service Provision (USP) program, which will then provide collective and individual access to basic telephony and internet services throughout the USA. The PAUSE mechanism proceeds as follows:

- Stage 1 is a simultaneous, multiple-round auction, conducted in three substages, with progressive eligibility requirements and an improvement margin requirement, with bidders submitting bids on individual properties; and
- Stage 2 is a simultaneous, multiple-round auction, conducted in substages, with progressive eligibility requirements and an improvement margin requirement, with composite bids to accommodate players' synergies.

Land, Powell & Steinberg (2006), generalize the PAUSE mechanism as a combinatorial, simultaneous, and progressive auction, with more than two stages. In fact, the auction may have as many stages as the number of lots/items to be auctioned, with each stage consisting of several bidding rounds.

Steinberg (2021) briefly discusses the application of the PAUSE mechanism to the auction of airport slots, during which the auctioneer aims to maximize the auction revenue. However, among other limitations, the properties of PAUSE are not clearly characterized in this context, where various other restrictions may be present, such as public service restrictions.

As it develops, the PAUSE procedure moves forward based on the monetary values of the proposed composite bids (a composite bid is a bid on a group of objects). We want to extend PAUSE to accommodate a richer value-function for the auctioneer, where a quality parameter is included alongside revenue.

The goal of allowing composite bids is to allow bidders to benefit from synergies between slots, or pairs of slots, in that the value a bidder places on a group of two or more slots may be higher than the sum of their individual values. In other words, a bidder value function exhibits 'super-additivity'. Allowing for combinatorial bids can also prevent the so-called 'exposure risk'. In the case of airport slots, it seems clear that one single slot without its pair is worthless, in that for any flight an airline will need a take-off slot at some airport and a landing slot at another airport. Therefore, the smallest group of airport slots with a positive value will be a pair of slots.

We can now look at four of the main PAUSE building blocks for auction design, in the context of airport slots.

The "product" to auction – it is easy to presume that the most elementary item to auction is "a slot" as defined in the EU (or in the US), in a specific level 3 airport. But this is a simplistic approach to the allocation problem. Apart from the concept of "right to a slot", which we believe should be a limited "temporal right" (perhaps between 10 and 20 years), it must also be acknowledged that when allocating slots, slot coordinators have to consider not only the runway capacity, but also terminal, arrival and departure pax (the number of passengers

carried by an airline) flow (Schengen and non-Schengen), passenger/cargo, as well as many other capacity or technical airport constraints. Therefore, far from just determining the number of available slots (for instance 10% of the airport slots) the auctioneer – that may be the slot coordinator – may have to associate not only a set of attributes related with all of those constraints to each of the slots available, but also take into consideration some public interest objectives such as those that are already being considered by slot coordinators today when applying slot allocation regulations, such as the ones pertaining to regional services, regular versus occasional services, etc.

This means that slots are not homogeneous, not only vis-à-vis the time frame they refer to, but also with regard to the same time period for the type of aircraft used, origin/destination served, service frequency, or even weekly pairing slots, depending on the (public) interest of the slot coordinator. We assume that the coordinator's objective function reflects all public interest issues regarding those slots.

The opportunity to auction origin/destination pairs of slots, whenever the same coordinator manages the slots in more than one coordinated airport should be considered, as is the case of Portugal for Lisbon, Oporto and Funchal airports[13]. It would be difficult to address origin/destination complementarity in a large territory (e.g., Europe) exclusively through an auction mechanism as slot allocation in all level 3 airports in this territory is performed in a decentralized manner (by local slot coordinators). Furthermore, this has been solved by the combination of "grandfather rights" and IATA Slot Conferences. However, we believe that the opportunity to simultaneously auction slots in all airports whose capacity is managed by the same slot coordinator should not be overlooked.

Therefore, the auctioneer could auction a portion (e.g., 10%) of all slots, from all the level 3 airports the auctioneer manages, but these slots could be paired with other airport slots or with slots in the same airport. They are also associated with a diverse set of constraints in relation to the attributes that the services provided by air transport operators must comply with. We refer to each of these as a "lot" to be auctioned, as a temporary right to use airport capacity to provide air transport services, given certain attributes and constraints.

The bidding variable – usually, the payment required for having the right to the items being auctioned is the bidding variable for all bidders, as the objective function of the auctioneer is assumed to be the maximization of the auction's revenue. Auction theory addresses the different properties of auction procedures, such as whether each auction procedure implements Pareto efficiency[14], but very often the auctioneer's objective is simply assumed to be the maximization of revenue.

Given the nature of airport lots and the air transport services they allow, on which public policy intervention is grounded, it is reasonable to assume that

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[13] Faro International Airport is a level 3 airport only during the summer season.

[14] More precisely, whether an auction procedure, as a mechanism, implements the Pareto correspondence.

the straightforward maximization of revenue might not be the auctioneer's sole objective, even though its efficiency effects associated with the alignment of capacity investment incentives cannot be ignored. In fact, the consideration of other public interest goals when allocating airport lots among airlines, other than the maximization of the auction's revenue, is clear in the existing regulations on slot allocation, as well as in WASG.

Most of the auction procedures proposed in the literature deal with these public interest objectives by assuming they will be reflected in the constraints imposed on how lots are to be used. Cardadeiro & Gata (2021) suggest that the use of a multidimensional objective function, where the auction revenue is only one of the dimensions, in line with the 'scoring auctions' literature. Furthermore, to define such objective function, they suggest the use of MACBETH[15] and Collaborative Value Modeling methodologies (Bana e Costa et al., 1994, 2012; Vieira et al., 2020) so that a robust objective function could be considered.

In this study we do not provide a concrete definition of such function, but we assume the existence of a two-dimension score function V (.,.), where the value for the auctioneer depends on the revenue obtained in the auction plus the value of a qualitative variable representing the public interest objectives that coordinators already take into consideration[16]. We denote this second variable by q, which stands for "service quality", or simply "quality". This score function V (.,.) is public information[17]. Each bidder i's bid for a particular item is a vector (m_i, q_i), where stands for monetary value, or price, the bidder is willing to pay for that item, and stands for the quality level the bidder guarantees, which the auctioneer values according to the score function V (. , .), and this fact is common knowledge. This extends the original PAUSE auction mechanism where the sole objective of the auctioneer is the maximization of revenue.

The inclusion of quality parameters does occur in many auction procedures, namely in many public tenders. Quite often minimum "quality" levels to be complied by bidders are set by the auctioneer. And bidders may still compete beyond such minimum levels, by offering higher quality levels, apart from competing in price as well. However, the case analyzed in this study is somewhat more complex, in particular as the type of items being auctioned off include single items and blocks of items.

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[15] The *Measuring Attractiveness by a Categorical Based Evaluation Technique* (or MACBETH) is a multicriteria decision analysis approach that allows for the quantification of the relative attractiveness of different options. This relative attractiveness can be qualitative based. MACBETH provides an approach to build a quantitative model of values based on qualitative (verbal) judgements, providing a path from ordinal to cardinal preference modelling.

[16] We assume the score function $V: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ to at least be (strictly) monotone in each of its two variables. More generously, we could assume the score function $V: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ to be differentiable and that $V_m(m,q) \geq 0$ and $V_q(m,q) \geq 0$ for all $(m,q) \in \mathbb{R}_+^2$. Furthermore, we assume it is not the case that for all $q_1, q_2, m_1, m_2 \in \mathbb{R}_+$, $V(m_1, q_1) > V(m_2, q_2)$ whenever $m_1 > m_2$.

[17] We are aware that making the score function public information facilitates collusion, even if the score function is multidimensional.

Combining auction revenues with other public interest aspects of lot allocation in the auctioneer's objective function (i.e., the score function) potentially leads to a final allocation that better balances the auctioneer's win objectives, when compared to the alternative where those non-monetary objectives are incorporated by setting minimum thresholds as constraints[18]. That balance will be evident throughout the auction, based on the tradeoffs implicit in the function.

However, there might be some instances where the introduction of constraints on usage of lots may be the best way of meeting public interest, as defined when the lots are auctioned. Note that, with this auction design, the public interest objectives promoted by the auctioneer can be addressed by a combination of the definition of the lots and the score value.

It is worth noting that the adoption of this score function will typically result in a lower auction revenue than would otherwise be obtained if the auctioneer's objective was to solely maximize revenue. This fact addresses the criticism of the undesirable competition effect of auctions and adds to all the other mitigation strategies proposed in the literature (Ball et al., 2018), increasing the social acceptability of auction mechanisms.

The bidder's constraints – the PAUSE auction mechanism, as generally described by Land, Powell & Steinberg (2006), runs for as many stages as the number of items to be auctioned which, among other properties, allows bidders to capture the synergies from combining in packages, or blocks, the items brought to the auction. However, in the airport lot allocation context, there are competition concerns that can limit bidders' market shares (i.e., airlines market shares), considering the lots being auctioned (e.g., 10% of the lots available) as well as the lots already held by the bidders (the remaining 90% of the lots). Therefore, the maximum number of stages could be less than the number of lots in the auction, and the auction might end at an earlier stage $\tau < |\Omega|$ if all bidders become inactive precisely due to market shares constraints. We assume each bidder knows from the start which maximum market shares he is subject to. Once the auction concludes, is it possible that a bidder ends up with market shares significantly below the upper limits he was subject to at the outset? As the auction proceeds, the bidder can get as close as possible to such upper limits through his bidding, as the winning bids at the end of each bidding round become public information. Hence, the bidder knows how much 'wiggle room' he has in the next bidding round, until the auction comes to an end. He can get as close as it is possible in

[18] Why did we choose to move the "minimum quality levels to be complied with" from the list of restrictions applied to the auctioneer's objective function? Because typically there is an information asymmetry between the auctioneer and the various bidders, with the latter holding an informational advantage over how far they can go in the provision of such quality levels. If we choose to place minimum quality levels to be complied with in the set of restrictions, the auctioneer will set those minimum levels according to the information he has collected to date and may set those levels below what airlines can accomplish. If, on the other hand, we include those quality parameters (or a single quality parameter, to begin simply) inside the auctioneer's objective function, we will make bidders compete for quality, and we could in the end obtain a higher equilibrium quality level.

this extended PAUSE auction to his market shares' upper limits, but he might very well end up below such limits.

Those market share constraints will probably impact bidders' bidding space heterogeneously, depending on the relevant markets in which each airline operates, but also on the number of lots each airline already holds. Those constraints are usually set based on an analysis by the relevant competition authorities but will undoubtedly distinguish incumbent airlines at the airport and new entrant airlines, as the existing regulations do for slots in the pool[19].

Whatever the competition constraints for each of the airlines admitted as bidders in the auction, the auctioneer can easily check each bid's compliance with each bidder's constraints, along with any other auction requirements regarding progressive eligibility or improvement margin requirements – as defined below. As combinatorial auctions are often electronically held, bid checking mechanisms must be introduced in the auction platform algorithms.

The "second-price" rule – given the properties of second-price auctions (Milgrom, 2021) most of the proposed lot auctions assume some type of second- price rule, which is not always a straightforward rule in combinatorial auctions (Day & Cramton, 2012). Land, Powell & Steinberg (2006) also assume a second- price payment scheme, but they use an example with only two items and two stages, and where the auctioneer's sole objective is the maximization of the auction's revenue.

In contrast, in our extended auction model, bids are two-dimensional, and each bid is associated with a score value. Therefore, the concept of second-price (payment scheme) must be revisited. We propose that at the end of the auction, each of the bidders that won any lot or block, must pay for each individual lot j won with the bid value $V_j^*(m_j^*, q_j^*)$ a value $\hat{m}_j$ equal to the score value of the second-best bid for that same lot, or block, i.e., $V_j'(m_j', q_j')$ or block, maintaining q_j^* fixed, that is $V_j(\hat{v}_j, q_j^*) = V_j'(m_j', q_j')$. The total amount each winning bidder has to pay is the sum in j of values $\hat{m}_j$ for all lots or blocks won by the bidder. This rule implies that the auctioneer will receive for each of the lots and blocks in the final winning bid - the amount that leaves him with the same score he would have obtained with the second-best score for the same lots and blocks.

Regarding the use of this revenue, we share Ball et al. (2018)'s approach which is to improve the acceptability of auction mechanisms by stakeholders, by reducing the landing fees, investing in local and regional air transportation systems and setting transparent revenue utilization rules announced in advance.

III. Extend PAUSE auction mechanism applied to the auction of airport lots

We extend the PAUSE auction mechanism by adapting Kelly & Steinberg's (2000), Land, Powell & Steinberg's findings (2006), and detailing Steinberg's work (2021) to the

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[19] In the European context, see Article 7 of Council Regulation (EEC) No. 95/93.

context of an auction involving airport lots, with the broad characteristics described in the previous section, together with a new auctioneer's objective function whose value depends on the auction's revenue and on "service quality", representing other desiderata such as the provision of a public service, a lower CO_2 emission and other types of emissions, to be defined at the outset.

Consider the discrete-time combinatorial auction procedure PAUSE, as described in the previous section, but applied to airport lots, following Steinberg's (2021). As explained before, it is a simultaneous ascending auction[20]. Let there be N bidders and let Ω denote the set of all airport lots to be auctioned. Let $\mathcal{P} = \{b_1, b_2, \dots, b_r\}$ denote a partition of set Ω[21]. Given partition $\mathcal{P}$ and a bidder i in N, a composite bid by this bidder i is a vector $\langle p(b_1), p(b_2), \dots, p(b_r) \rangle$ of (disjoint) block bids, which may contain some block bids from bidders other than j, such that $V_i(\mathcal{P}) = \sum_{j=1}^{r} V_i(b_j)$, where $V_i(b_j)$ is bidder i's bid for block b_j. A block can be a singleton, i.e., be composed of a single lot, or it can be a collection, or block, of different lots. The extended PAUSE auction proceeds in stages, with each stage possibly containing many auction rounds. The number of auction stages cannot exceed the number of all airport lots to be auctioned, i.e., the cardinality of set Ω[22]. The number of rounds in each stage will be defined once all bidding stops in that stage.

During each stage and auction round, each bidder will put forward a bid, which will always be a composite bid from stage 2 onwards, and whose value equals the value the two-dimension score function takes for the amount of money the bidder is willing to pay

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[20] A *simultaneous ascending auction* is an auction for multiple items, in which bidding occurs in rounds. In each round, bidders simultaneously make sealed bids for any items in which they are interested. After the bidding, round results are posted. The simultaneous ascending auction has proved to be a successful method of auctioning many related items. Simultaneous sale and ascending bids promote price discovery, which helps bidders build desirable packages of items. Although package bids are not allowed, the auction format does handle mild complementarities well. A simultaneous ascending auction is an effective mechanism for selling interdependent items (i.e., complements or substitutes). In a simultaneous ascending auction, and at any stage, a bidder is defined to be *active on a particular item* if either the bidder has the standing high bid from the previous round or submits an acceptable bid in the current round – see McAfee, R. P. & McMillan, J. (1996).

[21] Hence, $\bigcup_{j=1}^{r} b_j = \Omega$, $b_j \neq \emptyset$, and $b_i \cap b_j = \emptyset$ for all $i, j = 1, 2, \dots, r$, and $i \neq j$.

[22] Given the set Ω of all lots to be auctioned off, suppose any bidder (airline) i, with $i = 1, 2, \dots, N$, is required to submit his/her bids over every subset of Ω. Then, in a mechanism design context, his/her "economic environment" e_i would be defined by mapping from the power set $\mathbb{P}(\Omega) \setminus \emptyset$ into $\mathbb{R}_+^{|\mathbb{P}(\Omega)|-1}$, where $|\mathbb{P}(\Omega)|$ denotes the cardinality of power set $\mathbb{P}(\Omega)$, which equals $2^{|\Omega|}$. Note that due to synergies and complementarities, the value assigned to the union of two or more subsets of Ω in $\mathbb{P}(\Omega)$ might exceed the sum of the individual values assigned to each of those subsets. It may also be the case that because the bidder is not interested in certain subsets of $\mathbb{P}(\Omega)$, he/she assigns a value zero to them. If we consider the case where all lots are auctioned off through a sealed-bid private-value auction, each bidder will submit his/her bid for every element in $\mathbb{P}(\Omega)$. Or, as an alternative, he/she would submit bids only for the elements in $\mathbb{P}(\Omega)$ that he/she values positively; all the other elements in $\mathbb{P}(\Omega)$ would be assigned a value zero. Now, say $|\Omega| = 50$. Then, the bidder will have to assign a value to each of $|\mathbb{P}(\Omega)| - 1 = 2^{50} - 1$ possible (non-empty) subset of Ω. Therefore, it is not reasonable to define such a type of "economic environment" e_i for bidder i. The auction mechanism PAUSE avoids this dimensionality problem.

for the bid plus the "quality service" level included in that same bid.

According to PAUSE, in round 1 of stage 1, bidders (airlines) will decide whether to bid for the individual lots, and only individual lots, in Ω. Any bidder can submit bids for all or just for some individual lots (but will always be capped by market share constraints), including for just one lot or none. Round 1 terminates when all bids are collected, and the highest bid on each item (lot) is accepted by the auctioneer. All the highest bids are registered in a database by their respective owners, and it becomes public information[23]. Round 2 of stage 1 will take place only if one or more bidders decide to bid again (or for the first time), provided each bidder's renewed bid on any lot he bid in round 1 meets the "improvement margin requirement" ("imr"): a new bid over a lot must improve on the previous best bid on that lot (by the same or another bidder) by at least ε and strictly less than 2ε. The auctioneer will check whether the "imr" property is being met, using the two-dimension score function $V(.,.)..$ According to PAUSE, the justification for imr is to reduce the possibility of "price-jump bidding" and the "threshold problem", also known as "block-jump bidding". There are further rounds until all bidding ends during a particular round. Then stage 1 ends. Each item/lot is (temporarily) assigned to the bidder with the highest score. In other words, all highest bids are registered in the database of the respective bidders and are public information.

In stage 2, and as mentioned above, bidders (airlines) can submit bids on individual lots, as well as on packages of lots containing at most two lots. However, each airline must submit its bids as part of a composite bid, which contains a set of non-overlapping bids that cover all the lots in the auction, i.e., set Ω[24]. In other words, any composite bid is a partition of set Ω. Particularly important is the assumption that from stage 2 onwards, for the items/lots on which a bidder has no interest in bidding, he fills out his composite bid by making use of prior bids (i.e., bids from previous rounds) by any of the other bidders. This assumption is needed so that this bidder can submit a composite bid which, by definition, must include all items in Ω, either individually or in a package.

[23] It is worthwhile questioning if posting such information publicly among bidders, with bidders' identities being revealed side by side with their leading bids in each round/stage, will not facilitate collusion. See McAfee, R. P. & McMillan, J. (1996), pp. 169/170: *«A drawback of the simultaneous ascending auction is that collusion is easier than under a single round of sealed bids. Through their bidding patterns in the early rounds, the bidders might in effect say to each other: "This is 'my' territory. Stay away from it, and I will stay away from 'yours.' If you ever bid on it, I will drive up the price of 'yours' in retaliation". The FCC could have hindered collusion by revealing only the bid amounts between rounds, and not the bidders' identities. It chose not to do this in the broadband auction and instead gave out full details of each round's bidding, because it judged that the risk of collusion was outweighed by benefits of the information. (Bidder identities are useful to the bidders for evaluating the meaning of others' bids, reducing the winner's curse, and generally assisting sensible bidding) ».*

[24] More specifically, a composite bid includes: (i) a partition of the set Ω, (ii) a correspondence of each block included in a composite bid to a bidder and the price that bidder assigns to each of these blocks, and (iii) the value of the composite bid, defined as the sum of bids for each block considered individually, including blocks in which the bidder has no interest, but to which a value is assigned.

In each successive round of bidding, if a bidder intends to submit a new composite bid, then the imr is applied again, so that the new bid is valid. More specifically, if the number of blocks in the composite bid with new bids by the bidder is B, then the following applies, "the previous best composite bid + B.ε ≦ new composite bid by bidder < previous best bid + B.2ε". That is, the average increase in the bid score of each block in this bidder's composite bid will be at least B.ε, but strictly less than B.2ε. Stage 2 ends when all biddings end in a particular round. Throughout stage 2, the validity of each composite bid is verified by the auctioneer using his two-dimension score function V(. , .).

Each of the successive stages t, with t = 3,4,5, … , |Ω|, proceeds in the same way as stage 2. Bidders present their composite bids, containing individual lots and bundles of lots (i.e., blocks), but each one of these blocks can only contain up to t individual lots. In addition, each composite bid of each bidder corresponds to a partition of the set Ω. Each stage t can, again, contain several bidding rounds, without an a priori limit to the number of rounds. Each stage ends when bidding during that stage ceases. The final accepted composite bid in the auction displays the final lot allocation between airlines.

The following three diagrams A, B e C, depict three different situations that can arise when calculating the "second price", for each lot and each block within the final composite winning bid (i.e., the bid with the highest scoring value). They indicate what monetary values $\hat{m}_j$ the winners will have to pay for the lots or blocks assigned to each of them, and what quality levels q_j they must guarantee. Monetary values can be paid. But as it can be very difficult for a winning bidder (airline) to meet the "service quality" level included in the second highest bid – as, for example, when quality characteristics embedded in the second best bid, such as a larger airplane or a less-polluting airplane, are characteristics out of reach by the winner - the winner is only required to adjust his monetary payment m_j so that a final score equal the second-best score for each individual lot or block won. The second-best score is obtained by the pair (m'_j, q'_j) which lies on the iso-score line associated with this second-best score value V'.

Figure 1
"Second-score" rule

Panel A   Panel B   Panel C
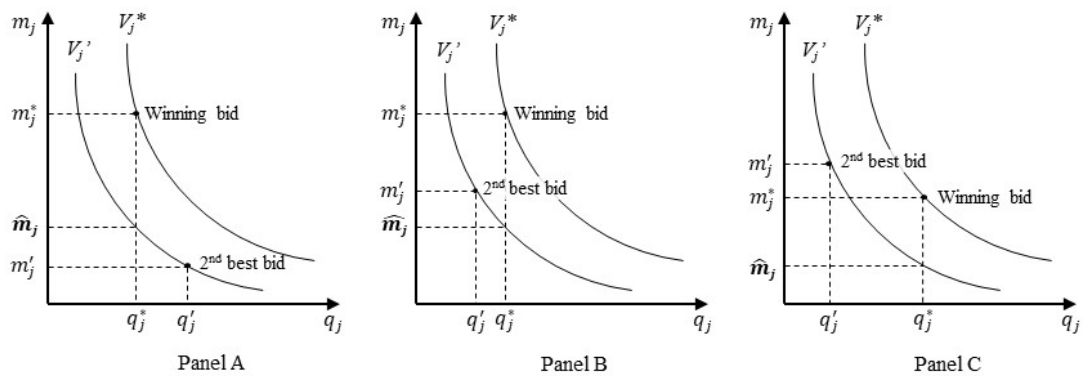
It is worth noting that the price $\widehat{m}_j$ paid for a specific lot or block by the winner of that lot or block, will always be lower than the offered price m_j^* by the winning bid, but might be higher (panel A) or lower (panels B and C) than the price offered by the bid with the second-highest score (m_j')[25]. The former situation will occur when the quality level of the winning bid is lower than the second-best bid, the latter when the quality is higher in the winning bid. In case of equal quality levels, $q_j^* = q_j'$, the winner will have to pay exactly the price offered by the second-best bid, $\widehat{m}_j = m_j'$.

Some of the main advantages of the PAUSE auction mechanism, as discussed by Steinberg et al. (2000, 2006 and 2021) still hold in this extended version. Namely: (a) Computability: Due to the introduction of composite bids by the PAUSE mechanism and its extended version, the choice of winning bids by the auctioneer is a computationally tractable problem[26]. Incidentally, the auctioneer may simply be a properly programmed algorithm. This is possible because the calculation effort, or computation, necessary to evaluate a packaged bid is transferred from the auctioneer to each of the various bidders. In this way, the auctioneer no longer faces the so-called "winner-determination problem"[27]; (b) Transparency: at each stage, and the end of each bidding round, each bidder can compare their bid with the winning bid to determine why they lost that round. In a bottom-up auction, such as PAUSE, this transparency promotes efficiency[28]; (c) Absence of Envy: in the final allocation of lots, no bidder will prefer to exchange the lots allocated to him with the lots allocated to any of the other bidders[29]; (d) Mitigation of the price-jump bidding problem: This is a potential problem in simultaneous auctions with multiple bidding rounds, where one or more bidders bid aggressively[30]. This form of

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[25] For the reasons cited above, we assume that when 'charged' the second-best, the winner of a lot or block will always keep unchanged the quality level q_j^* he submitted in his winning bid. He simply adjusts his offered price m_j^* downward to a level $\widehat{m}_j$ such that $V(\widehat{m}_j, q_j^*) = V(m_j', q_j')$.

[26] A problem is "computationally tractable" insofar as it is "computable in polynomial time and space", that is, in which the time (that is, the number of computational steps needed for any instance of the problem) and memory space occupied during computation are both polynomial functions of the "size" of the input (i.e., of the memory space occupied by the input, or the encoding length of that input, as an instance of the problem) that is "supplied" to the algorithm, for it to proceed with the computation of the output. Two basic references that can be referred to concerning computational complexity are: J. Hopcroft, *et al.* (2006) and Lewis, H. & Papadimitriou, C. (1981). See also: P. Cramton et al. (Eds.), 2006, pp. 303/4.

[27] Given a combinatorial auction, the '*winner-determination problem*' can be defined in a loose way as the problem of finding an allocation of items (lots and blocks) in the auction among all the bidders that will maximize the auctioneer's revenue. In our extended version, the auctioneer's objective is the maximization of his score function, and not just his revenue.

[28]   See McAfee, R. P. & J. McMillan (1996).

[29]Absence of envy is a possible criterion, among others, of "social justice", but it does not necessarily coincide with the Pareto efficiency criterion. Within a *'mechanism design'* approach, the mechanism underlying the extended PAUSE auction *implements* an envy-free 'social choice correspondence', where this correspondence maps the set of all admissible economic environments for all the bidders plus the auctioneer, to the set of all possible outcomes to all the bidders plus the auctioneer.

[30] See also McAfee, R. P. & J. McMillan (1996).

bidding consists of bidding well above the minimum required as a way of strategically sending a signal to other competing bidders not to compete for certain lots, given that the auction is characterized by 'private information' (about the value that each bidder assigns to each of the different batches). The problem can be mitigated by using the "improvement margin requirement", which we maintain in the extended PAUSE auction mechanism. "Block-jump bidding" is a related problem that can arise in combinatorial or composite auctions, as is the case with PAUSE; (e) Mitigation of the threshold problem: This problem is defined as the imbalance that may exist between large and small bidders, insofar as the introduction of the possibility of bidding on batch packages may favor large bidders seeking larger packages, because small bidders may have neither the incentive nor the ability to bid higher than high bidders. The fact that the auction is progressive, in the sense that the maximum size of the packages (lots) bid for during each stage also increases as each stage succeeds, until the conclusion of the auction, mitigates this problem; (f) Mitigation of the exposure problem: in the event that a bidder considers two or more lots to be complementary (which is obvious in the case of a lot including a departure "lot" at the airport of origin of the flight, without the corresponding "lot" of arrival/landing at the destination airport, or several pairs of "lots" that have a value greater than the sum of the value of each pair of "lots" because they exhibit synergies in the context of an airline's business plan). This problem is defined as the risk of winning some lots at auction and not the other lots that are complementary to them. The inclusion of bundles of lots, or blocks, in the auction mitigates this risk, allowing bidders to benefit from existing synergies between lots; and (g) Mitigation of the winner's curse problem: it means that the winner of an auction will pay a higher amount than the value that is lot is worth for the lot that he has won because he overestimates the value of the lot. PAUSE and its extended version mitigate this problem as its multiple stage and multiple round ascending design provide an effective price discovery mechanism, but also protection against winning lots without the combination bid. Furthermore, the transparency at each stage, and at the end of each bidding round, as well as the second-price rule, strengthens the mitigation properties of this auction.

IV. Applying the extended PAUSE auction mechanism to the three level 3 airports in Portugal

As previously mentioned, of the three international airports in mainland Portugal, two of them, Lisbon and Oporto airports, are classified as level 3 airports all year round. The other international airport, namely Faro airport, is classified as a level 3 airport only during the summer season, which is the high tourist season. Funchal International Airport, in the island of Madeira, is the other level 3 Portuguese airport classified at this level all year round.

   All these level 3 airports deal with congestion issues almost on a daily basis. Flight delays and sheer lack of capacity to meet the growing demand, have been persistent problems for many years. Apart from the recognized need to expand the airport capacity serving the Lisbon Metro Area[31], the use of a market mechanism, such as the described 'extended PAUSE auction mechanism" can help address these issues. As NERA (2004)

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[31] The Lisbon International Airport is the biggest and most important Portuguese airport, covering 134 destinations, operated by 55 scheduled airlines. It is one of the most important European gateways to Brazil and Africa and the biggest European airport serving South America among the Star Alliance hubs.

   See https://www.routesonline.com/airports/6453/ana-aeroportos-de-portugal-lisbon-airport.

claimed almost 20 years ago, when considering IATA's WASG, "the fact that existing airport charges fail to reflect the scarcity value of slots means that they may be allocated to services that are barely profitable at the current level of airport charges. Airlines that might be able to use these slots more efficiently and would therefore be willing to pay considerably more than the current level of charges, may nevertheless be unable to get hold of any slots". Our recommendation is that the above extended auction mechanism should be used in all of those three airports, in a gradual fashion, as suggested above.

V. Conclusions

In this paper, and following our previous work, we explore and extend the application of the PAUSE auction mechanism by defining a two-dimension 'score' function for the auctioneer, that includes another variable in addition to the total revenue. This additional variable can represent e.g., the quality level of the service provided by an airline.

We describe how this extended auction mechanism works in line with the work by Steinberg et al. (2000, 2006 and 2011). We suggest that this type of market mechanism be applied to three level 3 international airports operating in Portugal, on a very gradual basis, at the same time as the airlines using these three airports still follow the existing IATA WASG in their use of other airports, including the slot exchange protocols within those guidelines.

Some of the PAUSE auction mechanism's desirable properties, such as computability, transparency, absence of envy, and the mitigation of the "price- jump problem", "threshold problem", "exposure problem", and "winner's curse problem", still remain in place in the extended PAUSE mechanism.

Finally, adding to these properties, the inclusion of the "quality" variable in the objective function of the auctioneer transforms the auction into a "score bidding" instead of a "price bidding" auction, which we believe, along with the proposed second-price rule, will increase the acceptability of slot auctions by the industry's stakeholders.

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