Desynchronization for Decentralized Medium Access Control based on Gauss-Seidel Iterations

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Outline

Motivation

- Wireless Sensor Networks (WSNs) - a network composed of nodes using a wireless medium in Time Division Multiple Access (TDMA).
- No centralized infrastructure - implies the need for a decentralized algorithm to perform desynchronization of transmission.
- Applicable to surveillance - a group of vigilant robots that want to periodically visit sites to be monitored.

Traditional Solution

- A WSN can run Time-Synchronized Channel Hoping (TSCH) protocol established in IEEE 802.15.4e-2012 standard [1].
- Solution is inspired in biological agents modeled as Pulse-Coupled Oscillators (PCOs). In a sense similar to how fireflies adjust their firing rate depending on other fireflies.
- In [2] it is shown that with a minor change the algorithms based on PCOs can be seen as a gradient descent on a suitable quadratic function. It is proposed a Nesterov version to speed up convergence.

Intuition behind PCOs

- Assume an internal clock of each node that broadcast a pulse whenever its phase θ_i reaches 1, i.e., every T time units.
- Each nodes in the ring network adjusts its phase offset ϕ_i attempting to desynchronize from the others.
- Phase offsets are changed in a consensus-like iteration from the offsets of the two neighbors.
- $\theta_i(t) = \frac{t}{T} + \phi_i(t) \mod 1,$

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- Phase offsets are changed in a consensus-like iteration from the offsets of the two neighbors.
- $\phi_i^{(k)} = (1-\alpha)\phi_i^{(k-1)} + \frac{\alpha}{2}\left(\phi_{i-1}^{(k-1)} + \phi_{i+1}^{(k-1)}\right)$

Optimization formulation

- Shown in [2] that the PCO dynamics is equivalent to:

$$\phi^{(k)} = \phi^{(k-1)} - \frac{\alpha}{2}\nabla g(\phi^{(k-1)})$$

- This is the gradient descent algorithm applied to:

$$g(\phi) := \frac{1}{2}\|D\phi - \frac{1_n}{n} + \mathsf{e}_n\|_2^2$$

- Matrix D represents the network. Example for 4 nodes:

$$D = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

Problem Statement

- Find convergence rates for the optimization algorithms;
- Can we improve the speed by, instead, solving the linear equation;

$$\nabla g(\phi^\star) = 0 \tag{1}$$

- The algorithm should keep the sparse structure of the updates.

Desynchronization Problem

Does approaching the problem as a solution of a linear equation gets a faster algorithm?

- Yes following the concepts in [3].

Desynchronization using Gauss-Seidel (1/2)

- Since g is quadratic, its gradient is:

$$\nabla g(\phi) = D^\intercal D\phi + D^\intercal \mathsf{e}_n.$$

- Convert into the format $Ax = b$ by taking $A = D^\intercal D$ and $b = -D^\intercal \mathsf{e}_n$.
- Partitioning $A = L + D + U$, for lower and upper triangular matrices L and U and diagonal D;
- The Gauss-Seidel Method becomes:

$$x(k+1) = (L+D)^{-1}(b - Ux(k)) \qquad (2)$$

Desynchronization using Gauss-Seidel (2/2)

- The method can be written to take advantage of updated values

$$x_i(k+1) = \frac{1}{A_{ii}} \left(b_i - \sum_{j=1}^{i-1} A_{ij} x_j(k+1) - \sum_{j=i+1}^{n} A_{ij} x_j(k) \right). \quad (3)$$

- In this context becomes:

$$\phi_1^{(k+1)} = \frac{1}{2}\left(1 - \phi_2^{(k)} - \phi_n^{(k)}\right)$$

$$\phi_i^{(k+1)} = \frac{1}{2}\left(-\phi_{i-1}^{(k+1)} - \phi_{i+1}^{(k)}\right), 2 \leq i \leq n-1$$

$$\phi_n^{(k)} = \frac{1}{2}\left(-1 - \phi_1^{(k+1)} - \phi_{n-1}^{(k+1)}\right)$$

Results for the optimization

- Leveraging writing the GRADIENT, NESTEROV and HEAVY-BALL as linear dynamical systems we show theoretical convergence rates.
- Since matrix $Q = D^\intercal D$ is symmetric and circulant, it is possible to compute explicitly its real eigenvalues $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_n$.
- Worst-case convergence rate only depends on λ_2 and λ_n.
- In the journal version, these results are extended for the optimal fixed parameter selection.
- The convergence rate for the Nesterov version proposed in [2] is also computed and compared against the simpler fixed parameter version.

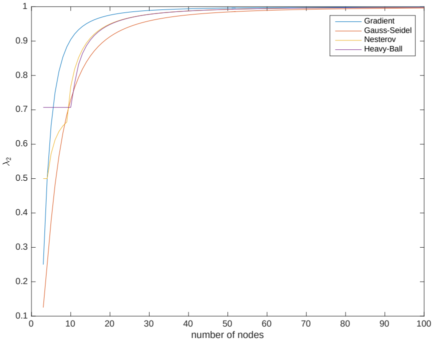
Results for the Gauss-Seidel version

- Theoretical result: the Gauss-Seidel version is convergent, i.e., $|\lambda_2(T_{GS})| < 1$ and $\forall i : |\lambda_i(T_{GS})| \leq 1$;
- The transition matrix is given as a finite-sum of matrices.
- In the journal version it is provided relaxed versions of the algorithms and a comparison with the optimization methods.

Simulation Results (1/2)

Setup: Fixing the gradient step to be $1/\max \lambda_i(Q)$ and setting the momentum term to $1/2$.

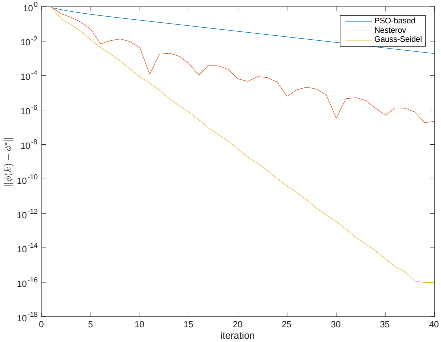
- Worst-case convergence rate as a function of n.
- Using these parameters, PCO is clearly slower.
- HEAVY-BALL has a smaller convergence rate but very similar to NESTEROV.
- GS achieves a faster convergence as n increases.



Simulation Results (2/2)

Setup: First simulated $n = 5$ and then $n = 20$ both for GRADIENT, NESTEROV, HEAVY-BALL and GS.

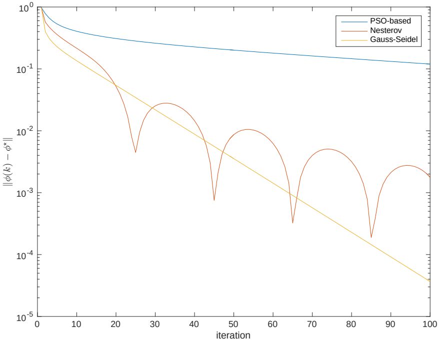
- As expected PCO is much slower;
- The proposed NESTEROV in [2] has a oscillating error;
- In smaller networks GS has a exponential decrease in the error.



Simulation Results (2/2)

Setup: First simulated $n = 5$ and then $n = 20$ both for GRADIENT, NESTEROV, HEAVY-BALL and GS.

- Increasing n emphasizes the difference between PCO and NESTEROV;
- The oscillation effect gets larger;
- GS still has a steady exponential decrease.



Concluding Remarks

Contributions:

- We have shown theoretical convergence rates for the optimization algorithms and the Gauss-Seidel version for the desynchronization problem.
- Gauss-Seidel always has exponential convergence, requires no parameters and is distributed.
- In the journal version, we show that indeed the choice in the literature for the parameters of the Nesterov method can be improved.
- Additional convergence rates both for optimal fixed parameters (known n) or time varying (unknown n).

References

IEEE, "IEEE Standard for Local and metropolitan area networks–Part 15.4: Low-Rate Wireless Personal Area Networks (LR-WPANs) Amendment 1: MAC sublayer," *IEEE Std 802.15.4e-2012*, pp. 1–225, 2012.

N. Deligiannis, J. F. C. Mota, G. Smart, and Y. Andreopoulos, "Fast desynchronization for decentralized multichannel medium access control," *IEEE Transactions on Communications*, vol. 63, no. 9, pp. 3336–3349, 2015.

D. Silvestre, J. Hespanha, and C. Silvestre, "A pagerank algorithm based on asynchronous gauss-seidel iterations," in *Annual American Control Conference (ACC)*, 2018, pp. 484–489. DOI: 10.23919/ACC.2018.8431212.

The end

- Thank you for your time.

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