

PAPER • OPEN ACCESS

Enhancing teaching and learning of fluid mechanics with interactive computational modelling

To cite this article: R G Neves 2019 *J. Phys.: Conf. Ser.* **1286** 012047

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the [collection](#) - download the first chapter of every title for free.

Enhancing teaching and learning of fluid mechanics with interactive computational modelling

R G Neves^{1,2}

¹ *Departamento de Ciências e Tecnologias (DCT) e Autónoma TechLab – Centro de Investigação em Tecnologias, Universidade Autónoma de Lisboa (UAL), Rua de Santa Marta 47, 6º Andar, 1150-293 Lisboa, Portugal*

² *Unidade de Investigação Educação e Desenvolvimento (UIED), Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa (FCT/UNL), Monte da Caparica, 2829-516 Caparica, Portugal*

¹rneves@autonoma.pt ²rgn@fct.unl.pt

Abstract. Contemporary science, technology, engineering and mathematics (STEM) increasingly require advanced knowledge about mathematical physics models and methods of scientific computation. In STEM educational environments this entails the necessity to implement pedagogical curricula and methodologies that epistemologically balance the inclusion of interactive engagement sequences of computational modelling activities, and offer students opportunities to develop meaningful knowledge of physics, mathematics and scientific computation, as well as of the specific STEM concepts and processes. Our approach in this context is based on interactive engagement activities built around computational modelling experiments implemented in the Modellus environment that span the range of different kinds of modelling, from exploratory to expressive modelling. In this paper we describe research concerning a sequence of activities about hydrostatic pressure forces and torques, a theme of an introductory fluid mechanics course for undergraduate university engineering students having only elementary knowledge of secondary education physics and mathematics and no significant prior knowledge about scientific computation. We analyse student's perceptions about the activities and the effects generated on the learning process. Using a Likert scale questionnaire, we show that students reacted positively to the Modellus based activities, considering them helpful in the learning process of the mathematics and physics of fluid mechanics and for their overall professional training. The results also show that students considered Modellus a useful software for computational activities that help the learning of mathematical physics models, sufficiently easy to learn and user-friendly. Based on the analysis of student's work content, we show that students were able to construct and explore the proposed mathematical physics models and simulations, and establish meaningful and operationally reified relations with the relevant hydrostatic phenomena. We also show that the computational modelling activities were effective in resolving several difficulties persisting after theoretical lectures and problem-solving paper and pen activities.

1. Introduction

Science, technology, engineering and mathematics (STEM) are profoundly interrelated structures of knowledge. On one hand, science evolves from the progressive development of hypotheses, questions and models, generators of theories, which have a strong mathematical character as scientific reasoning, concepts and laws are represented by mathematical reasoning, entities and relations. On the other hand,



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

scientific explanations and predictions must be consistent with the results of systematic and reliable experiments, which depend on technological or industrial developments as much as these depend on the progress of science and mathematics [1-3].

In this context, the development of STEM knowledge results from the interaction of individual and group actions where modelling involves an epistemological balance between theoretical, experimental and computational elements. Such modelling processes unfold in cycles, each passing through different cognitive phases, namely, qualitative contextualization, definition, exploration, interpretation and validation of the relevant mathematical models, communication of modelling results and the development of generalizations. Across all cycle phases, computational modelling has been playing an increasingly important role in enhancing calculation, exploration and visualization capabilities. Being root subjects for the development of knowledge in other STEM subjects, physics and mathematics are particular yet fundamental examples in this context. In physics and mathematics, knowledge is based on rigorous declarative and procedural specifications of abstract concepts and of their interrelations, and in all epistemological and cognitive phases, it requires operational familiarization and reification, high theoretical and methodological consistency and a precise relation with the relevant referents, either in real natural or techno-industrial phenomena or in abstract mathematical worlds [1-3].

This range of epistemological and cognitive features is still not reflected by the majority of current introductory physics and mathematics courses. For example, in university education the traditional general physics courses of the first two or three years are organized around expositive theoretical lectures, and recipe based problem-solving and experimental laboratory classes. In general, these are courses that cover superficially a large number of physics topics, and limit the application of computational methods and technologies to the presentation of text, images or simulations, or to a supporting role in data acquisition and analysis. Usually, today as in the past, these courses are found difficult and disappointing, and have low exam success rates. The continuing evidence is for the development of a highly fragmented and inconsistent knowledge of physics and mathematics [4-6] and for average expectations that decrease over time [7].

However, many research efforts have shown that this situation can be improved if interactive engagement learning environments approximately recreating the modelling processes of research are implemented [8-12]. In various settings, student's learning processes have been shown to be more meaningful, leading to better knowledge development and structuring, as well as better resolution of cognitive conflicts with erroneous common sense beliefs or incorrect physical and mathematical conceptions. It thus remains important to reinforce the work along these research lines and invest on the implementation of the research results in actual educational practice.

A crucial aspect in this context is the role of computational modelling methods and technologies. It is well known that in a growing number of STEM areas modelling actions depend on knowledge about advanced computational mathematical physics models. The corresponding learning processes should then involve the progressive development of strong backgrounds in physics, mathematics and programming. As a consequence, at the introductory level, from secondary education to the first two or three years of university education, physics and mathematics learning environments in STEM education should epistemologically balance the integration of interactive engagement sequences of computational modelling activities.

This integration should occur with a stepwise introduction of the necessary elements of scientific computation, in order to control the extra cognitive load associated with the process of learning operational programming and software knowledge. Such control is difficult to achieve with programming languages like Fortran [13], Pascal [14], Java [15] or Python [16], scientific computation software like Mathematica or Matlab, or even with educational programming languages like Logo [17] or Boxer [18], since these systems end up requiring that students acquire too much programming and software knowledge alongside knowledge about physics and mathematics. To reduce the cognitive load and create more favourable conditions for exploratory and expressive modelling, several computer modelling systems have been developed, e.g., Coach [19], Easy Java Simulations and Physlet [20], Modellus [1-3, 21-24] and the PhET simulations [25].

Our approach in this context has involved the integration of interactive engagement learning activities built around computational modelling experiments implemented in the Modellus environment (see, e.g., [1-3, 21-24]). In previous research work we have shown that Modellus is a useful versatile system which allows the creation of exploratory and expressive computational modelling activities to teach introductory physics and mathematics, while achieving an effective control of the amount of programming and software knowledge that must be introduced. Several sequences of activities were created for Newtonian particle mechanics, electromagnetism and blackbody radiation laws applications. Most students felt very favourably about the inclusion of the Modellus based activities, which were able to help them address several cognitive conflicts in the understanding of physics and mathematical concepts, explore different representations of mathematical models, analyse the interplay between analytical and numerical approaches and consider more realistic problems at an earlier learning stage.

In this paper we consider the application of our interactive computational modelling approach to an introductory fluid mechanics course we offered to a group of first year undergraduate university students enrolled in engineering majors. In this context, we describe research concerning an illustrative sequence of learning activities about hydrostatic pressure forces and torques.

2. Teaching organization and methodology

We start with a brief description of the main aspects related to the teaching organization and methodology used for an effective application of our approach. The introductory fluid mechanics course was the first part of a Fluids and Thermodynamics course we taught to a class of 150 students from university majors in automation and control engineering, chemical engineering, civil engineering, electric engineering, environmental engineering, food engineering, mechanical engineering and production engineering at Univas. These students had only an elementary knowledge of secondary education physics and mathematics, quite fragmented and based on a small number of memorized formulas, and no significant prior knowledge about scientific computation, only user knowledge of daily life informatics.

The course was organized in 3 separate class groups, each with around 50 students. For each of these classes, the course involved a flexible time sequence of 3 complementary components, namely, theoretical lectures, paper and pen problem-solving lessons, and computational modelling activities with Modellus. To create an interactive engagement environment in all components students were organized in group teams of 3 to 5 students. All class components were taught in rooms with individual student tables that during group work could be easily moved into more convenient configurations, for example, forming a circle to facilitate collaborative discussions. For the computational modelling activities students used their own laptop computers, 1 or 2 per group.

After the definition of the relevant theoretical framework, complemented with a set of worked out examples, always presented in an atmosphere open for free questioning and discussion, the student groups were oriented to work on sequences of paper and pen problem-solving activities and computational modelling activities with Modellus, composed by a selection of problems having clear contact with easily observed phenomena and increasing difficulty levels. Class resources included PDF documents with the paper and pen activity problems and a summary of the theory given in the lectures, the Modellus package examples and the set of PDF documents containing instructions to build the Modellus mathematical models, animations, graphs and tables required to solve the proposed computational modelling activities. Both in the paper and pen problem-solving activities and in the computational modelling activities the student groups were motivated to analyse, discuss and solve the activity problems on their own, using the suggested course bibliography and the class resources made available in class or through the Moodle platform. During the exploration of the activities, the teams were never left working alone but continuously helped to ensure adequate working rhythms with appropriate conceptual, analytical and computational understanding. Whenever felt necessary, global class discussions were conducted to keep the pace, to introduce new themes, or to clarify any doubts on concepts, reasoning or calculations common to several teams.

The implemented assessment procedures involved continuous group and individual evaluation based on the regular class activities, the work reports of the paper and pen problem-solving activity assignments, the work reports of the Modellus computational modelling activity assignments, and the in class individual written tests. At the end of the course, students free willingly answered an anonymous Likert scale questionnaire, not counting for the final grade, to evaluate their perceptions about the integration of Modellus and the associated interactive computational modelling activities.

3. Fluid mechanics with interactive computational modelling: An illustrative learning sequence

The fluid mechanics part of the Fluids and Thermodynamics course introduced students to the standard elements of fluid statics and dynamics, covering density, pressure forces, pressure variation with depth and Pascal law, buoyancy and Arquimedes principle, superficial tension, continuity and Bernoulli equations, Venturi effect and Torricelli law, viscosity and turbulence. Each sequence of learning activities involved 3 interconnected modelling phases, namely, (1) theoretical framing, (2) paper and pen problem-solving, and (3) computational modelling problem-solving. An illustrative learning sequence in this context considered the phenomenological setting of a water dam and the analysis of the associated hydrostatic pressure forces and torques. The complete learning sequence involved two parts. The first part introduced the resultant pressure force as a vector and determined its magnitude for a dam with a vertical rectangular wall. The second part analysed the resultant pressure force torques and determined the centre of pressure, the application point of the resultant pressure force on the rectangular wall. This learning sequence was then generalized to consider the extension of the analysis to other surface geometries and inclinations, for example a submerged circular gate located in the inclined wall of a water reservoir. In what follows we describe the research referring to the first part of the implemented hydrostatic force learning sequence.

The theoretical framing was built from a background of basic secondary education particle mechanics and previously introduced knowledge defining fluid, density, pressure forces and static equilibrium. In the first stage of theoretical framing, a deduction of the law defining the variation of pressure with depth was discussed. In many instances, the water in the dam can be considered an incompressible fluid in static equilibrium, where the pressure increases with depth because at a greater depth it is necessary to support the weight of a larger amount of matter. For a small cylindrical column of height h that is in static equilibrium inside an incompressible fluid with density ρ (for water $\rho = 1.0 \times 10^3 \text{ kg/m}^3$), the pressure P at the bottom of the column is equal to the pressure exerted at the bottom by the fluid column itself plus the pressure P_t exerted at the top of the column, $P = P_t + \rho gh$, where $g = 9.8 \text{ m/s}^2$ is the magnitude of the acceleration of gravity. Consequently, for an incompressible fluid in static equilibrium, the pressure increases linearly with depth and is always the same at the same depth.

The second stage of theoretical framing discussed a deduction of the hydrostatic pressure force applied on the wall of the dam. The simplified modelling setting of a vertical rectangular wall with length L that is sustaining a body of water with depth h_f was considered. Using graphics to help with visualization students were then conducted through the following mathematical physics reasoning. Take the orthogonal reference frame $Oxyz$, where Oz is perpendicular to the wall and points to the water, Oy marks the depth y , and Ox marks the length along the wall. Then divide the wall in a large number N of very small surface strip elements each one with area $\Delta A = L\Delta y$. This partition defines the Oy sequence $y_0 = 0, y_1, \dots, y_N = h_f$ where $\Delta y = y_{j+1} - y_j, j = 0, \dots, N - 1$. The limit of this partition is $N \rightarrow +\infty, \Delta y \rightarrow 0$. The pressure $P(y)$ on the surface element ΔA located at depth y is $P(y) = \rho gy$, where we have not explicitly written the pressure variations inside the surface element which are at most proportional to Δy . Hence, the hydrostatic pressure force applied on this surface element has magnitude $\Delta F = \rho gyL\Delta y$ and is perpendicular to the surface element pointing away from the water, $\overline{\Delta F} = -\Delta F\vec{u}_z$, where \vec{u}_z is the unitary vector of Oz . The resultant pressure force applied on the wall is obtained summing all the elementary forces applied to all the surface elements, taking at the end the limit $N \rightarrow +\infty, \Delta y \rightarrow 0$. Since all elementary vectors have the same direction this force is given by $\vec{F} = -F\vec{u}_z$ where

$$F = F(h_f) = \rho g L \lim_{\substack{N \rightarrow +\infty \\ \Delta y \rightarrow 0}} \sum_{j=0}^{N-1} y_j (y_{j+1} - y_j) = \rho g L \lim_{\substack{N \rightarrow +\infty \\ \Delta y \rightarrow 0}} \frac{1}{2} \sum_{j=0}^{N-1} (y_{j+1} + y_j) (y_{j+1} - y_j). \quad (1)$$

The limit sums in equation (1) are equivalent Riemann sums defining the integral $\int_0^{h_f} y \, dy$. They can be explicitly calculated, leading to the exact analytical result $F = F_{\text{analytical}} = \rho g L h_f^2 / 2$.

The paper and pen problem-solving phase was developed in two stages of group work: (1) Discussion and explanation of the theoretical framework leading to $F = \rho g L h_f^2 / 2$, and (2) Application of the deduced formula to solve the following example problem: The resultant pressure force the water exerts on the rectangular inner wall of a dam has magnitude 3.0×10^5 kN. If the length of the wall is 50 m what is the depth of the mass of water the dam is sustaining?

Content analysis of student's work reports showed that only 28% of the 150 students attained fairly well both (1) and (2). This was clearly a consequence of student's lack of understanding of the abstract and subtle mathematical physics reasoning developed to deduce and calculate the Riemann sums defining the resultant pressure force \vec{F} . The computational modelling activities were then designed to make the Riemann summing process and associated limits more concrete and thus improve understanding. The integration is then numerical applying the trapezoidal rule to define an iterative process that implements the Riemann sums depending on an adjustable finite step. In this iterative numerical solution the next value of F , for instance $F(y + \Delta y)$, is equal to the last known value of F , $F(y)$, plus the rate of change of F , $L\rho g y$, times the iteration step Δy : $F(y + \Delta y) = F(y) + L\rho g y \Delta y$. Given a starting value, e.g. $F(0) = 0$, it is possible to determine $F(y)$ going through a succession of step Δy iterations. The numerical value of the Riemann integral in equation (1) is then given by the accumulated value $F(h_f)$. This numerical method linearizes the problem in the interval $[y, y + \Delta y]$ and thus produces a solution that approximates the exact analytical solution $F_{\text{analytical}} = \rho g L h_f^2 / 2$. Since the method converges, a smaller iteration step will produce a better approximation, that is, the approximation error given by $\varepsilon_F = |F - F_{\text{analytical}}| / F_{\text{analytical}}$ will decrease with the iteration step Δy . However, the smaller the iteration step the larger the number of iterations and, consequently, the computation time.

The group activities of the computational modelling problem-solving phase started with a discussion of this numerical theoretical framing. The next step was to build the Modellus numerical models, understand their functioning in the context of the analytical and numerical theoretical framing, and solve a set of proposed example problems.

The starting problem was the following: A dam with a rectangular inner wall of length $L = 40$ m sustains water up to a depth of 30 m.

- a) What is the magnitude of the resultant pressure force exerted on the wall when the water depth is $h_f = 30$ m? Compare the numerical solutions obtained with steps $\Delta y = 1.0$ m and $\Delta y = 0.10$ m with the analytical solution and discuss the results.
- b) What is the magnitude of the resultant pressure force when the water depth is $h_f = 10$ m and $h_f = 20$ m?
- c) What happens if the length of the inner wall of the dam is $L = 50$ m?

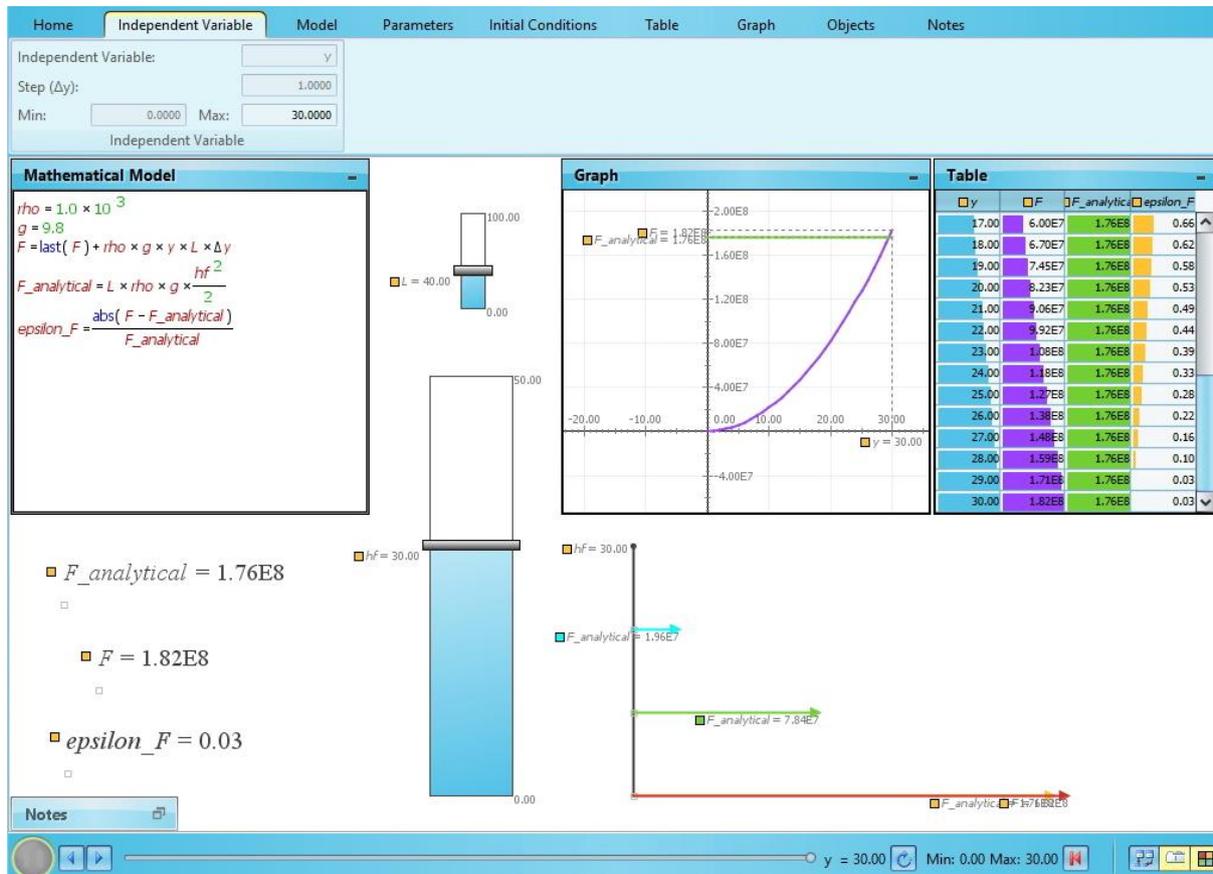


Figure 1. Modellus model to determine the magnitude of the resultant pressure force applied on a dam with a rectangular inner wall of length L which sustains a body of water with depth h_f . For $L = 40$ m and $h_f = 30$ m, the integration with step $\Delta y = 1.00$ m leads to $F = 1.82 \times 10^5$ kN, $F_{\text{analytical}} = 1.76 \times 10^5$ kN, and $\epsilon_F = 0.03$, and the integration with step $\Delta y = 0.100$ m leads to $F = 1.770 \times 10^5$ kN, $F_{\text{analytical}} = 1.764 \times 10^5$ kN, and $\epsilon_F = 0.003$.

To solve this problem the student groups built a Modellus model (figure 1) starting with the Mathematical Model. The magnitude of the acceleration of gravity g and the water density ρ are introduced as fixed parameters in the Mathematical Model and the water depth y is defined as the Independent Variable to which a range interval, e.g. $[0, 30]$ m, and a numerical step Δy , e.g. $\Delta y = 1.0$ m, are associated. The numerical integration leading to F is then programmed in the Mathematical Model using $F = \text{last}(F) + L\rho g y \Delta y$, with $F = 0$ N as initial condition. The last two lines in the Mathematical model define $F_{\text{analytical}}$ and ϵ_F as dependent variables. The length of the wall L and the water depth h_f are independent variables which are defined as free adjustable parameters in the model. For the proposed problem a total of 6 Cases need to be created. The Animation is then built with 2 Level Indicators, one for L and another for h_f , 3 Variables to visualize F , $F_{\text{analytical}}$ and ϵ_F , a Geometric Object to represent the dam wall, and 4 Vectors to represent \vec{F} and $\vec{F}_{\text{analytical}}$. The adjustment of the Animation configuration is done with the Animation grid. The Table can also be defined to visualize y , F , $F_{\text{analytical}}$ and ϵ_F . In the Graph, $F(y)$ can be plotted showing, alongside the growing Vector \vec{F} , the Table and the Variable F , the build-up of the Riemann sum as the successive iterative terms add up. Also clearly visible in the Graph is the fact that the magnitude of the pressure force F increases with the square of the water depth y . The process of taking the Riemann limits $N \rightarrow +\infty$, $\Delta y \rightarrow 0$ is made explicit and concrete by running the model for decreasing numerical iterative steps Δy . This also shows that the

numerical method is convergent since ϵ_F decreases with Δy . The increase in computation time is also clearly observable.

A follow up problem was the following: The maximum resultant pressure force a dam with a rectangular inner wall of length $L = 50$ m can withstand has magnitude $F_f = 5.0 \times 10^5$ kN.

- a) What is the maximum depth of water the dam can sustain? Compare the numerical solutions obtained with steps $\Delta F = 1.0 \times 10^6$ N and $\Delta F = 1.0 \times 10^5$ N with the analytical solution and discuss the results.
- b) What happens if the length of the inner wall of the dam is $L = 60$ m? And if $F_f = 7.0 \times 10^5$ kN?

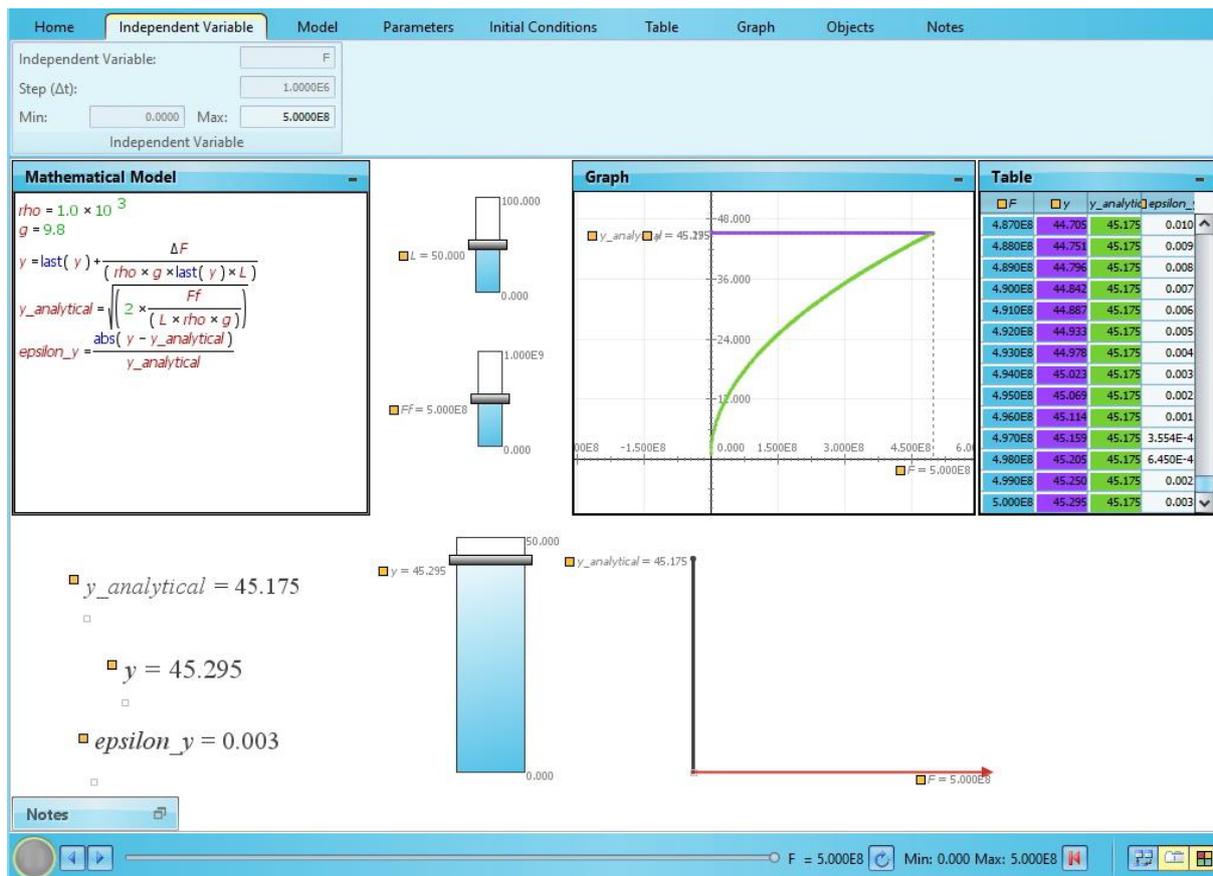


Figure 2. Modellus model to determine the maximum depth of water a dam with a rectangular inner wall of length L can sustain if it is built to support at most a resultant pressure force with magnitude F_f . For $L = 50$ m and $F_f = 5.000 \times 10^5$ kN, the integration with step $\Delta F = 1.000 \times 10^3$ kN leads to $y = 45.295$ m, $y_{\text{analytical}} = 45.175$ m, and $\epsilon_y = 0.003$, and the integration with step $\Delta F = 1.000 \times 10^2$ kN leads to $y = 45.195$ m, $y_{\text{analytical}} = 45.175$ m, and $\epsilon_y = 4.382 \times 10^{-4}$.

The corresponding Modellus model is depicted in figure 2. The Independent Variable is now the magnitude of the pressure force F , e.g. with the range interval $[0, 5.0 \times 10^8]$ N and the numerical step $\Delta F = 1.0 \times 10^6$ N. The depth y is obtained by numerical integration, $y = \text{last}(y) + \Delta F / (L \rho g \times \text{last}(y))$, using $y = 1$ m as the initial condition. The free adjustable parameters in this model are the independent variables L and F_f . The proposed problem then requires the creation of 4 Cases. In the Animation the Level Indicator for y , the Vector \vec{F} , or the Graph again allow a clear visualization of the Riemann summing process and how the water depth increases when the magnitude of the pressure force

increases. The value of y corresponding to $F_f = 5.0 \times 10^5$ kN is the maximum depth of water the dam can sustain.

After this sequence of activities the content analysis of the student's work reports showed that 70% of the 150 students were able to build the proposed mathematical physics models and animations, understand fairly well their functioning and theoretical framing, and solve the proposed problems.

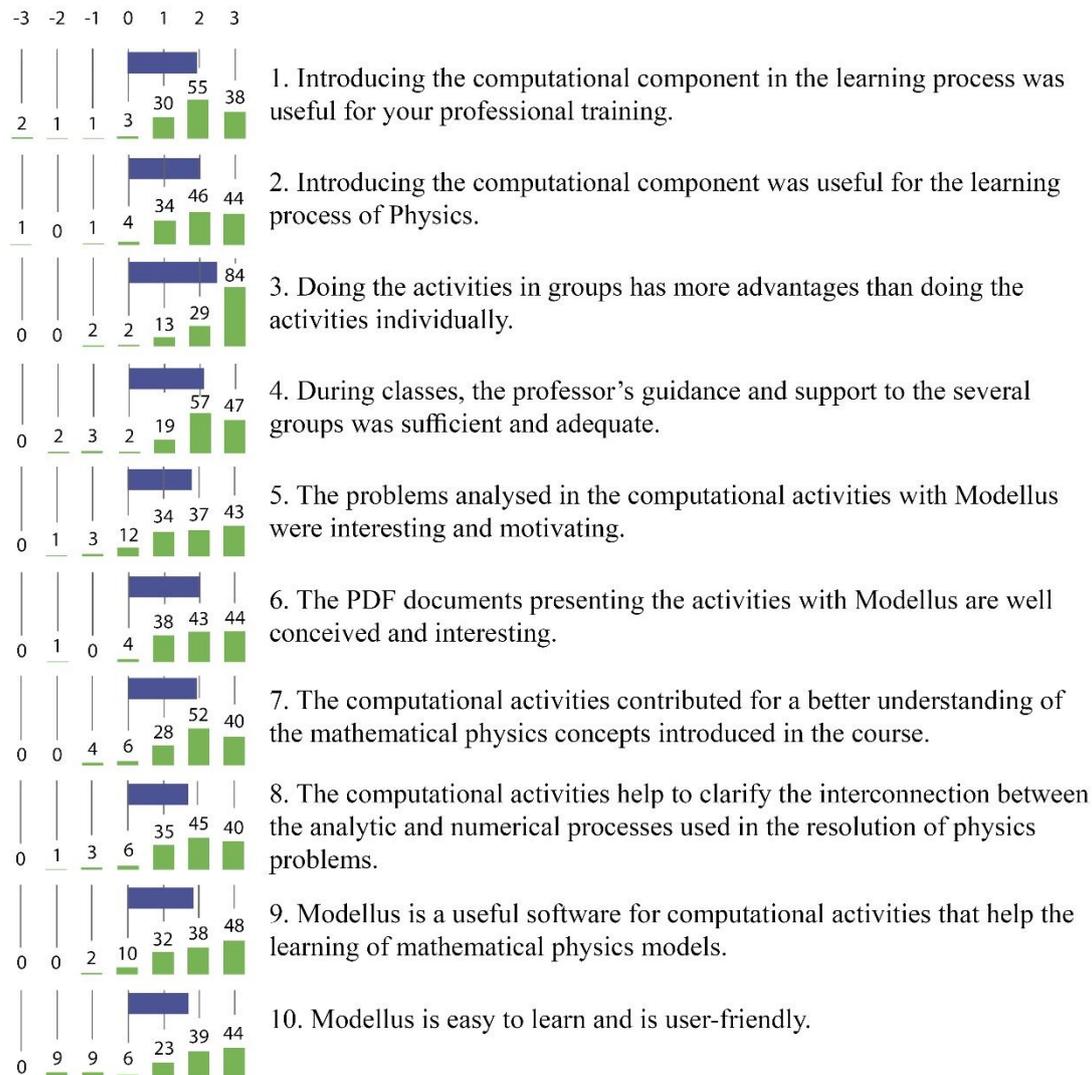


Figure 3. Introductory fluid mechanics student opinion questionnaire and results. For each questionnaire statement the Likert scale starts at -3 and ends at +3, -3 stating complete disagreement, +3 complete agreement and 0 no preferred opinion. The bar graph (blue) shows the distribution over the Likert scale of the average student opinion per questionnaire statement. The bar graph (green) shows the distribution over the Likert scale of the number of students that answered the questionnaire.

4. Conclusions

Let us first note that the results of the Likert scale questionnaires show that the majority of students felt very positively about the interactive computational modelling activities with Modellus (figure 3). Defining the average opinion of a student as the average over all answers given by the student to the questionnaire statements, the results show that 97.7% of the students gave a positive opinion, averaging 1 (22.3%), 2 (44.6%) or 3 (30.8%), 1.5% averaged no preferred opinion, and 0.8% gave a negative

opinion. In the Likert scale, the average opinion of all students was 2.0. On the other hand, using the average opinion of all students relative to each one of the questionnaire's statements, we find that students showed clear preference for working in interactive collaborative groups, with adequate guidance and support. Most students considered the computational modelling activities useful for the learning process of the concepts and mathematical physics models of physics, in particular of fluid mechanics, as well as for their overall professional training. They also considered the activities helpful in clarifying the interplay between the analytical and numerical processes used to solve problems in physics. Less unanimously, Modellus was considered sufficiently helpful and user friendly, and the PDF documents interesting and motivating.

On the other hand, content analysis of student's work reports referring to the learning sequence about hydrostatic pressure forces shows: (1) After the first two phases, theoretical framing and paper and pen problem-solving, most students (72%) were unable to understand the Riemann summing process and associated limits involved in determining the resultant pressure force; (2) The computational modelling activities contributed successfully to resolve student's difficulties and strengthen student's knowledge about these matters, as more students (70%) were now able to construct and explore the pressure force mathematical physics models and animations, understand their functioning in the context of the analytical and numerical theoretical framing, clearly establishing more meaningful and operationally reified relations with the hydrostatic phenomenon in the concrete setting of a water dam.

These results show that the computational modelling activities with Modellus were very well received by the students, allowing them to clearly improve the knowledge acquired in the first two phases of the learning sequence, resolve several identified learning difficulties, and significantly expand the knowledge horizon about hydrostatic pressure forces. With the computational modelling activities the fluid mechanics course thus demonstrated enhanced capacity and efficacy to amplify the knowledge of students in a structured and consistent way. In addition, Modellus has once more proved itself a useful system to create explorative to expressive computational modelling activities for introductory physics and mathematics education. Modellus allows a stepwise introduction to scientific computation, with a fruitful interplay between analytical and numerical methods used to solve problems in physics and mathematics, as well as an efficient control of the level of programming and specific software overhead that needs to be introduced. Again, this success is rooted in the capability Modellus has of allowing the simultaneous manipulation and analysis of several different model representations, namely, tables, graphs and animations with interactive objects having properties defined in a visible and modifiable mathematical physics model.

Although the class implementation of the computational modelling activities was successful and the results obtained positive, in answers to questions during class dialogues students manifested some caution about the process of learning physics with Modellus and computational methods, mainly because for them it meant extra work to master computational modelling besides just physics and mathematics. Students also felt that the content load was heavy for the time available to complete the sequence of learning activities.

Future work will involve, e.g., (1) Development of new learning sequences, both in classical physics and in contemporary physics, creating new sets of interactive digital documentation and software resources, (2) Implementation of field research actions to test and improve the new sequences and resources, analysing the corresponding learning processes, and (3) Field research actions to compare Modellus and other computer modelling systems, including programming languages, as computational modelling instruments to create interactive computational modelling activities.

Acknowledgments

Work partially supported by CEU-UAL/Autónoma TechLab and UIED/FCT/UNL.

References

- [1] Neves R, Silva J and Teodoro V 2011 *Improving learning in science and mathematics with exploratory and interactive computational modelling ICTMA14 - Trends in Teaching and*

- Learning of Mathematical Modelling* (International Perspectives on the Teaching and Learning of Mathematical Modelling vol 1) ed G Kaiser, R Borromeo-Ferri and G Stillman (Dordrecht: Springer) chapter 33 pp 331-341 DOI: 10.1007/978-94-007-0910-2_33
- [2] Neves R, Silva J and Teodoro V 2013 *Integrating computational modelling in science, technology, engineering and mathematics education Educational Interfaces between Mathematics and Industry: Report on an ICMI-ICIAM-Study* (New ICMI Study Series vol 16) ed A Damlamian, J Rodrigues and R Straesser (Dordrecht: Springer) chapter 38 pp 375-383 DOI: 10.1007/978-3-319-02270-3_38
- [3] Neves R and Teodoro V 2013 *Revista Lusófona de Educação* **25** 35
URL: <http://revistas.ulusofona.pt/index.php/reducacao/article/view/4379>
- [4] Halloun I and Hestenes D 1985 *Am. J. Phys.* **53** 1043 DOI: 10.1119/1.14030
- [5] Halloun I and Hestenes D 1985 *Am. J. Phys.* **53** 1056 DOI: 10.1119/1.14031
- [6] McDermott L 1991 *Am. J. Phys.* **59** 301 DOI: 10.1119/1.16539
- [7] Redish E, Saul J and Steinberg R 1998 *Am. J. Phys.* **66** 212 DOI: 10.1119/1.18847
- [8] McDermott L and Redish E 1999 *Am. J. Phys.* **67** 755 DOI: 10.1119/1.19122
- [9] Handelsman J *et al* 2005 *Science* **304** 521 DOI: 10.1126/science.1096022
- [10] van den Berg E, Ellermeijer T and Slooten O (Editors) 2006 *Proc. GIREP Conference 2006: Modeling in Physics and Physics Education* (Amsterdam: University of Amsterdam) ISBN: 9789057761775
- [11] Blum W, Galbraith P, Henn H-W and Niss M (Editors) 2007 *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (New York: Springer) DOI: 10.1007/978-0-387-29822-1
- [12] Meltzer D and Thornton R 2012 *Am. J. Phys.* **80** 478 DOI: 10.1119/1.3678299
- [13] Bork A 1967 *Fortran for Physics* (Reading: Addison-Wesley) ASIN: B007T53W18
- [14] Redish E and Wilson J 1993 *Am. J. Phys.* **61** 222 DOI: 10.1119/1.17295
- [15] Gould H, Tobochnik J and Christian W 2007 *An Introduction to Computer Simulation Methods: Applications to Physical Systems* (San Francisco: Addison-Wesley) ISBN: 9780805377583
- [16] Chabay R and Sherwood B 2008 *Am. J. Phys.* **76** 307 DOI: 10.1119/1.2835054
- [17] Papert S 1980 *Mindstorms: Children, Computers and Powerful Ideas* (New York: Basic Books) ISBN: 0465046274
- [18] di Sessa A 2000 *Changing Minds: Computers, Learning and Literacy* (Cambridge: MIT Press) ISBN: 9780262041805
- [19] Heck A, Kadziarska E and Ellermeijer T 2009 *J. Computers in Mathematics and Science Teaching* **28** 147 URL: <https://www.learntechlib.org/primary/p/30302>
- [20] Christian W and Esquembre F 2007 *Phys. Teach.* **45** 475 DOI: 10.1119/1.2798358
- [21] Teodoro V and Neves R 2011 *Computer Physics Communications* **182** 8 DOI: 10.1016/j.cpc.2010.05.021
- [22] Neves R, Neves M C and Teodoro V 2013 *Computers & Geosciences* **56** 119 DOI: 10.1016/j.cageo.2013.03.010
- [23] Neves R, Schwartz J, Silva J, Teodoro V and Vieira P 2012 *Learning introductory physics with computational modelling and interactive environments Proc. of the GIREP-EPEC 2011 Conference - Physics Alive* ed A Lindell, A-L Kahkonen and J Viiri (Jyvaskyla: University of Jyvaskyla) pp. 208-213 URL: <https://jyx.jyu.fi/handle/123456789/49276>
- [24] Neves R 2017 *Revista Lusófona de Educação* **35** 171
URL: <http://revistas.ulusofona.pt/index.php/reducacao/article/view/5921>
- [25] Wieman C, Perkins K and Adams W 2008 *Am. J. Phys.* **76** 393 DOI: 10.1119/1.2815365