Phase Drift Estimation and Symbol Detection in Digital Communications: A Stochastic Recursive Filtering Approach

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Abstract—This paper proposes a novel Bayesian stochastic filtering approach for the simultaneous phase drift estimation and symbol detection in digital communications. The posterior density of the phase drift is propagated in a recursive fashion by implementing a prediction and a filtering step in each iteration. The prediction step is supported on a random walk model playing the role of prior for the phase drift process; the filtering step is supported on a Gaussian sum approximation for the probability density of the current observation, i.e., the so-called sensor factor. The Gaussian sum approximation turns out to be the key element allowing to derive a fast and efficient stochastic filter, which otherwise would be very hard to compute. The detection of the digital symbols is then carried out based on the inferred statistics of the phase drift. The effectiveness of the proposed method is illustrated for BPSK signals in the presence of strong phase drift.

*Index Terms*—Stochastic recursive filtering; Gaussian sum filter; phase drift; state estimation; burst communications.

## I. Introduction

In digital communications, the lack of phase synchronization between the receiver oscillator and the received signal often renders communication systems useless. Estimating the phase drift, which includes phase noise due to oscillators instabilities and frequency offset caused by Doppler effect and/or poor frequency alignment between oscillators, is of utmost relevance.

Under a Bayesian perspective, the phase drift estimation from the observation of the channel output amounts to determining the posterior probability density function (pdf) of the state (*i.e.*, the current phase drift) conditioned on all measurement data, thus providing the means to compute an optimal estimate with respect to any criterion, *e.g.*, minimum meansquared error (MMSE). The determination of the posterior pdf is usually extremely difficult. A remarkable exception occurs when the state and the measurement equations are linear and the noise is additive and Gaussian distributed. In this case, the posterior pdf is efficiently computed by the well known Kalman filter.

In this paper, we introduce a stochastic recursive filter to propagate the posterior pdf based on two central features:

1) the phase drift is modeled by a random walk model playing the role of prior.

2) the sensor factor is approximated by a sum of Gaussian functions.

These two features allow to exploit well known properties of Gaussian functions to derive an efficient and effective stochastic filter, in a vein similar to that of the Gaussian sum filter introduced in [1]. The detection of the digital symbols is then carried out based on the inferred statistics of the phase drift.

To illustrate the effectiveness of the proposed filter, we compare the performance results obtained when modeling the pdf by a weighted sum of Gaussian functions with that achieved using a single Gaussian. We assume perfect channel estimation and M-phase shift-keying (M-PSK) modulation with perfect symbol synchronization.

The paper is organized in the following manner. Section II establishes the models for the channel output and for the phase drift. Section III defines the Gaussian sum filter, with subsections dedicated to the Gaussian sum approximation, the Gaussian fitting of the sensor factor, the evaluation of the filtering and prediction densities, the filter initialization, and the implementation aspects of the algorithm. Section IV is devoted to the symbol detection, while Section V presents the performance results and Sec. VI concludes the paper.

## II. SYSTEM MODEL

Consider the transmission of the BPSK symbols  $\{s_n = e^{j\phi_n}; n = 0, \dots, N-1\}$ , where  $\phi_n \in \{0, \pi\}$ , over an additive white Gaussian noise (AWGN) in the presence of phase drift. The received signal at the detector output is given by

$$y_n = e^{j(\phi_n + \theta_n)} + v_n; \qquad n = 0, \dots, N - 1.$$
 (1)

In (1),  $\{v_n; n=0,\ldots,N-1\}$  is a sequence of complex zeromean white Gaussian noise random variables with uncorrelated real and imaginary components each one with variance  $\sigma_v^2 = N_0/(2E_b)$ , where  $E_b$  stands for the bit energy.

The phase drift  $\theta_n$  is modeled by a random walk process described by the linear stochastic difference equation

$$\theta_n = \theta_{n-1} + w_n; \qquad n = 0, \dots, N-1,$$
 (2)

where  $\{w_n; n=0,\ldots,N-1\}$  is a sequence of real zero-mean independent Gaussian random variables with variance  $\sigma_w^2$ .