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Efficient Uniform Channel Quantization of Sparse CIR for Downlink OFDM Systems

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Abstract Channel state information at the transmitter side is an important issue for wireless communications systems, namely when precoding techniques are employed. Recent works explored random vector quantization (RVQ) as a solution for limited feedback for multi-user systems equipped with multiple antennas. Despite of being a good option for narrowband channels, this method requires large complexity and is not efficient for sparse channels. To overcome these drawbacks we consider a strategy based on uniform quantization, denoted partial uniform quantization (P-UQ), where just part of channel frequency response is quantized. This allows an efficient feedback of channel frequency response from the receivers to the transmitter, by using a reduced number of quantization bits. The comparison between the proposed P-UQ-based method and RVQ performed in this paper leads to the conclusion that the most advantageous method for sparse channels is the P-UQ.

Keywords Channel state information · Limited feedback · Random vector quantization · Sparse channels · Uniform quantization

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1 Introduction

Multiple-input multiple-output (MIMO) wireless channels exploit antenna arrays at both the transmitters and receivers, increasing the demanding capacity and quality of wireless communication links [1–3]. For cellular systems with multiple transmit antennas at the base station (BS) and multiple users with one or more receive antennas, referred to as MIMO broadcast channel [4], it is possible to achieve high capacity by coordinating the transmissions to multiple users simultaneously [5, 6].

Precoding is a solution to improve performances of multi-antenna multi-user based systems, by filtering the transmitted data streams by a precoding matrix at the transmitter, usually computed based on knowledge of the full channel state information at the transmitter (CSIT) [7–9]. The knowledge of CSIT is absolutely crucial to achieve such improvements and assuming perfect CSIT is not realistic in many practical scenarios. In frequency-division duplex systems, full channel state information must be conveyed through a feedback channel [10]. This is impractical when the channel is severely time-dispersive (i.e., highly selective in the frequency) since the number of channel coefficients that need to be quantized and sent back to the transmitter is high, leading to large overhead penalty. To overcome this drawback, recent studies addressed the issue of limited feedback, where the CSI is quantized and fed back to the transmitter through a limited link [11–13].

Limited feedback was first introduced in [14, 15], where to limit the overhead needed to feed back CSIT it was considered that receiver and transmitter maintain a common beamformer codebook. Codebook design can be obtained through maximizing the average signal-to-noise ratio (SNR) at the maximum ratio combining output and by applying Loyd algorithm for vector quantization [16]. For the particular case of Rayleigh fading channel, the codebook design can be seen as a sphere vector quantization problem, known as Grassmannian line packing [17]. The complexity of quantized feedback increases with codebook size and large Grassmannian codebooks are difficult to design and encode. Moreover it cannot be applied to the systems where the CSI exhibits no special structure, as the main case of interest of multi-user multi-antenna systems, where the CSI to be fed back is a set of channel matrices [18]. Random vector quantization (RVQ) codebooks, firstly defined in [19, 20], are used in these cases because the optimal vector quantizer for this problem is not known in general [21, 22]. RVQ is a simple approach of codebook design that generates the vectors independently from a uniform distribution on the complex unit sphere. The beamforming expressions for RVQ limited feedback in terms of average bit error probability and ergodic capacity on a MISO (multi-input single-output) system are derived in [23]. Although RVQ techniques allow efficient multi-antenna multi-user schemes with limited feedback, the required codebooks can be very large, especially when we have a high number of transmit and receive antennas. Another major drawback of RVQ is computational complexity [24]. In that case, a question arises: whether it is preferable to employ RVQ or simpler quantizers, working on a sample-by-sample basis, as in [25, 26]. The answer to this question, as the development of efficient feedback strategies with reduced overhead is crucial to improve multi-user multi-antenna transmissions.

The channel for broadband wireless communications is usually sparse. A discrete signal is sparse in some domain such as time, frequency, or space if most of the samples are zero. For example a channel has sparsity L in time if only L taps have large values while the other taps with minor values can be neglected (i.e. the weight of CIR is L) [27, 28]. Sparsity in channel has been exploited in different fields in communications by reducing sampling rates and processing manipulations in coding, spectral estimation, array processing, component analysis, and multipath channel estimation [29].

Motivated by RVQ limitations for MIMO sparse channels, where a large overhead is required to achieve acceptable performances, we propose a new limited feedback strategy based on uniform quantization (UQ) referred to as partial UQ (P-UQ), which allows efficient CSI feedback for multi-user MIMO–OFDM based systems. We also compare the performance and the complexity of the proposed method with the RVQ technique. The P-UQ strategy reduces the number of quantization bits while maintaining performances close to the RVQ. With the proposed limited feedback strategy the sparse characteristic of the broadband channels is explored, where overhead scales with the multipath sparsity of the channel. A quantized version of the CSI associated with the different links between a BS and user terminals (UTs) is fed back from the UTs to the BS. In the proposed method just the non-zero taps of CIR are quantized and fed back, followed by a reconstruction of the channel frequency response (CFR), thus reducing the quantization overhead. Moreover the P-UQ method has much lower complexity than the RVQ based technique, since it does not require the use of large codebooks.

The remainder of the paper is organized as follows: Sect. 2 presents the multi-user MISO system model for multicell OFDM systems. The channel quantization strategies for limited feedback are presented in Sect. 3. Section 4 presents the main simulation results. The main conclusions are drawn in Sect. 5.

Notation Bold-face capital letters denote matrices, bold-face lowercase letters denote column vectors. The symbol $\tilde{\cdot}$ is used to represent the channel in the time domain, while the variables representing the channel without the tilde are in the frequency domain; indexes n and l are used in time and frequency domains, respectively; indexes m and k refer to the BS antenna and the user, respectively. The operation $(\cdot)^H$ represents the Hermitian transpose of a matrix; $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the real and imaginary parts of a complex number, respectively; j is the imaginary unit; and, $\mathrm{tr}(\cdot)$ refers to the trace of a square matrix. $\mathrm{DFT}[\cdot]$ refers to the discrete Fourier transform and $\mathrm{IDFT}[\cdot]$ to the inverse discrete Fourier transform.

2 System Characterization

In this paper we consider the downlink of a cellular OFDM-based system. We assume one BS equipped with M antennas transmitting to K single antenna UTs sharing the same resources, as shown in Fig. 1, with $M > K$. An OFDM modulation with N available subcarriers is employed at each BS and linear precoding is done individually over the N subcarriers. The CFR associated to the link between the BS and the kth UT for the lth subcarrier is represented by the vector $\mathbf{h}_{k,l} \in \mathbb{C}^{M\times 1}$. The quantized version of the CSI is fed back using one of the quantization strategies described in detail in the next section.

Channel impulse response (CIR) sample of order n for the ith OFDM symbol, with $i \in \mathbb{N}$, is represented by the matrix

$$\tilde{\mathbf{H}}_n^{(i)} = \left[\tilde{\mathbf{h}}_{1,n}^{(i)}\, \tilde{\mathbf{h}}_{2,n}^{(i)} \ldots \tilde{\mathbf{h}}_{K,n}^{(i)}\right]^T, \tag{1}$$

where $\tilde{\mathbf{h}}_{k,n}^{(i)}$ refers to the nth samples of CIR between BS and kth UT and $\tilde{\mathbf{h}}_{k,n}^{(i)} \in \mathbb{C}^{M\times 1}$; each matrix component is expressed as $\tilde{h}_{k,m,n}^{(i)} = \tilde{\mathbf{H}}_n^{(i)}(k,m)$ and is given by

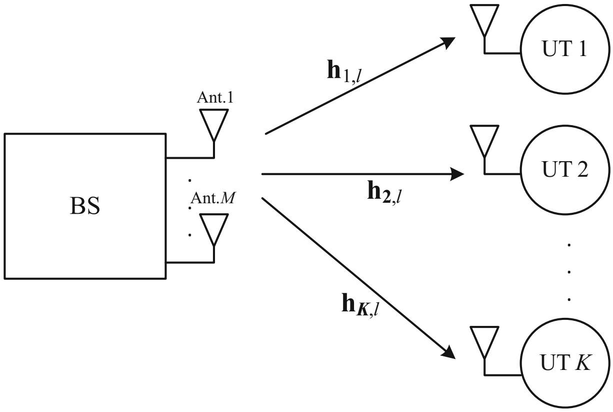

Fig. 1 Generic block diagram of the considered MISO multi-user system

$\tilde{h}^{(i)}_{k,m,n} = \tilde{h}^{BB}_{k,m}(n\Delta t_s + iT_s)$; $\Delta t_s = \frac{T_s}{N}$, T_s is the duration of the OFDM block, $\tilde{h}^{BB}_{k,m}(t) = \sum_{p=0}^{L-1} \beta_{k,m,p}\delta\left(t - \tau_{k,m,p}\right)$ is the complex baseband representation of the CIR between the mth BS antenna and the kth UT; L is the number of paths for such link; $\beta_{k,m,p}$ is the complex amplitude of the pth path and $\tau_{k,m,p}$ is the delay of the pth path. Of these N CIR components, just L are non-zero, with $L \ll N$ for sparse channels. The channel admits a small number of multipath components in multiples of sample period, where symbol has a long interval comparatively to the duration of the channel [28].

The CFR for subcarrier l is defined as $\mathbf{H}^{(i)}_l = [\mathbf{h}^{(i)}_{1,l}\,\mathbf{h}^{(i)}_{2,l}\ldots\mathbf{h}^{(i)}_{K,l}]^T$, with $\mathbf{H}^{(i)}_l \in \mathbb{C}^{K\times M}$ and it is related with the CIR through

$$\left\{h^{(i)}_{k,m,l}; l = 0, 1, \ldots, N-1\right\} = \mathrm{DFT}\left[\left\{\tilde{h}^{(i)}_{k,m,n}; n = 0, 1, \ldots, N-1\right\}\right]. \tag{2}$$

The CFR between the BS and the UT k for the ith OFDM symbol is characterized by the sequence of vectors $\left\{\mathbf{h}^{(i)}_{k,l}; l = 0, 1, \ldots, N-1\right\}$.

Under linear precoding, the received signals, in frequency domain, on the lth subcarrier ($l = 0, \ldots, N-1$) are given by

$$\mathbf{y}_l = \mathbf{H}_l\mathbf{W}_l\mathbf{s}_l + \mathbf{n}_l, \tag{3}$$

where $\mathbf{H}_l = [\mathbf{h}_{1,l}\,\mathbf{h}_{2,l}\ldots\mathbf{h}_{K,l}]^T$, with $\mathbf{H}_l \in \mathbb{C}^{K\times M}$, is the equivalent channel that contains the flat Rayleigh fading coefficients with i.i.d. $\mathcal{CN}(0,1)$ entries; $\mathbf{s}_l$ is the data symbols vector, with $\mathbf{s}_l \in \mathbb{C}^{K\times 1}$ and $\mathrm{E}\left[\mathbf{s}_l\mathbf{s}_l^H\right] = \mathbf{I}_K$; $\mathbf{W}_l \in \mathbb{C}^{M\times K}$ is the linear precoding matrix computed at the BS on subcarrier l, calculated by

$$\mathbf{W}_l = \alpha\left(\mathbf{H}^Q_l\right)^H\left(\mathbf{H}^Q_l\left(\mathbf{H}^Q_l\right)^H + \sigma_n^2\mathbf{I}_K\right)^{-1}, \tag{4}$$

where $\mathbf{H}^Q_l$ is the equivalent channel after quantization, for subcarrier l; $\mathbf{n}_l$ is the additive white Gaussian noise (AWGN) vector at subcarrier l, i.e., $\mathbf{n}_l \sim \mathcal{CN}\left(0, \sigma_n^2\mathbf{I}_K\right)$ and α is a normalization factor such that $\mathrm{tr}\left(\mathbf{W}_l^H\mathbf{W}_l\right) = 1$.

3 Channel Quantization Strategy

In this section we describe two quantization methods that are used to feed back the CSI from the receivers to the transmitter. The CFR is estimated at the receiver through appropriate training sequences and/or pilots [30]. The estimated channel is then quantized and sent to the BS to compute the precoding matrix defined in (4).

3.1 Random Vector Quantization Method

In this sub-section we briefly describe the RVQ feedback quantization technique often considered for MIMO based systems, which is used here for comparison purposes [24, 29]. For the sake of simplicity, we will drop the dependence on the OFDM symbol index. The constructed codebook for channel direction information (CDI), defined as the normalized CSI (i.e., $\mathbf{h}_{k,l}^{d} = \mathbf{h}_{k,l} / \left|\mathbf{h}_{k,l}\right|$), is formed by 2^B vectors i.i.d. on the M-dimensional unit sphere, $\{\mathbf{c}_b\}, b = 1, \ldots, 2^B$, where B represents the number of feedback bits per OFDM symbol and user. Each user quantizes its CDI to a codeword in a given codebook $\mathbf{C}_k \in \mathbb{C}^{M \times 2^B}$ and the codebook is predetermined and known at both the BS and user sides. Partial CSI is acquired at the transmitter via a finite rate feedback channel from each of the receivers. Furthermore we use the minimum Euclidean distance to choose the codeword closest to each channel vector direction, i.e.,

$$f_{k,l} = \arg \min_{i=1,\ldots,2^{B_1}} \left\| \mathbf{h}_{k,l}^{d} - \mathbf{c}_i \right\|^2, \tag{5}$$

with $k = 1, \ldots, K$, $l = 0, 1, \ldots, N - 1$. Thus, after each UT having sent the index of the codeword to the BS, the BS obtain the CSI through the corresponding codebooks and using the indexes given by $\mathbf{h}_{k,l}^{Q} = \mathbf{c}_{f_{k,l}}$, so that it can design the precoder matrices [21]. Only the CDI is sent, dismissing the channel magnitude information with this method [24].

3.2 Uniform Quantization Method

In this sub-section we describe the proposed efficient channel quantization procedure based on UQ. Assuming severely time-dispersive channels the RVQ based schemes requires the quantization of N samples. To reduce it, for the special case of sparse channels, we propose a method referred to as P-UQ, where just the non-zero taps of the CIR are quantized and fed back to the transmitter, and then the CFR is recovered [31]. The CIR has a duration that must be smaller than the duration of the cyclic prefix, N_{CP} (notice that the referred duration is measured in terms of number of samples), which for typical OFDM implementations is much lower than N. For the case of sparse channel just L taps of the N_{CP} samples are non-zero, with $L \ll N_{CP} \ll N$. Just these samples are chosen to be quantized $\left\{\tilde{h}_{k,m,n}; n \in \Psi\right\}$ (it should be pointed out that in the samples $\tilde{h}_{k,m,n}$ the index n is not necessarily associated to a given multipath component when the multipath components are not symbol-spaced), where Ψ is the set of sample positions that correspond to the L delay paths that are non zero, then obtaining $\left\{\tilde{h}_{k,m,n}^{Q}; n \in \Psi\right\}$ considering the separate quantization of the real and imaginary parts of each of the appropriate samples as following

$$\tilde{h}^{Q}_{k,m,n} = f_Q\left(\mathrm{Re}\left\{\tilde{h}_{k,m,n}\right\}\right) + jf_Q\left(\mathrm{Im}\left\{\tilde{h}_{k,m,n}\right\}\right), \tag{6}$$

where $f_Q(\cdot)$ denotes the quantization characteristic. The CIR after quantization becomes

$$\tilde{h}^{Qz}_{k,m,n} = \begin{cases} \tilde{h}^{Q}_{k,m,n}, & \text{if } n \in \Psi \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The reconstructed CFR is obtained through the DFT of the CIR, with the reconstructed CFR sequence being expressed as

$$\left\{h^{R}_{k,m,l}, l = 0, 1, \ldots, N-1\right\} = DFT\left\{\tilde{h}^{R2}_{k,m,n}, n = 0, 1, \ldots, N\right\}. \tag{8}$$

If we consider to use b quantization bits for each real and imaginary parts of the quantized sample for UQ and B bits for each user and subcarrier for RVQ, thus the number of bits required for the total CSI quantization of an OFDM frame per user with UQ is $2bMN$ and with RVQ is BN bits. For the proposed P-UQ method the number of bits is hugely reduced to $2bML \ll 2bMN$. The reduction will be of N_s/N comparatively to the RVQ (considering the both schemes with the same quantization overhead). For example for the case of $N = 1024$ and $L = 6$ it needs less than 1% of the overheads. For comparison purposes we can also consider an equivalent method for RVQ, where just part of CFR samples are quantized (the same number as the samples quantized in UQ), and the corresponding number of quantization bits is also reduced to BKL. However, as observed in the next section, in this case the partial RVQ will have large penalties in comparison with the proposed method. Table 1 summarizes the quantization overhead of the referred methods: RVQ and P-UQ and P-RVQ (quantization of just part of the channel samples).

4 Performance Results

In this section we present a set of performance results for the quantization techniques described above. Our scenario has a BS equipped with $M = 2$ antennas and $K = 2$ single antenna UTs. However, the conclusions would be similar for other antenna and UTs configuration. The main parameters used in the simulations are based on long term evolution (LTE) standard [32]: FFT size of 1024; sampling frequency set to 15.36 MHz; subcarrier separation is 15 kHz and modulation is QPSK. We considered two different sparse channels. For channel 1 we have a uniform PDP with 64 taps equally spaced with uncorrelated Rayleigh fading. Channel 2 is a sparse channel based on pedestrian ITU BRAN B channel, with just 6 taps with irregular delay space between them according to

Table 1 Number of quantization bits required for the discussed quantization strategies for one OFDM block: RVQ and UQ (quantization of all the CFR samples), P-RVQ and P-UQ (quantization of just part of the samples)

Quantization strategy	Number of quantization bits per block
RVQ	BKN
P-RVQ	BKL
UQ	$2bKMN$
P-UQ	$2bKML$

[33]. For simplification purposes, we consider that delays taps of CIR are multiples of sampling period. We assume perfect CSI estimation at receiver side (i.e., at the UTs) and that feedback channel is considered to be error-free and with no delay.

In UQ we consider 2^b levels and an optimum normalized saturation level (which depends on the channel) and for the RVQ we use B bits for each user. In order to compare both strategies we assume to have the same total number of quantization bits in Figs. 4, 5 and 6, thus resulting in $B = 2bM$. The comparison between the quantization strategies is made for the cases of quantization of all the CFR samples for both RVQ and UQ methods. Moreover we also perform the proposed method with partial sample quantization of CFR (P-CFR) in order to fairly compare with the RVQ we simulate it in similar conditions, just quantizing the same amount of samples than in the P-UQ.

Figures 2 and 3 show the impact of the normalized saturation level A_M/σ and the number of quantization b on SQNR, for channels 1 and 2, respectively. As can be observed, there is an optimum normalized saturation level for each value of b, since the quantizer's saturation becomes too frequent if A_M/σ is small and the quantization interval becomes too high when A_M/σ is high. Hereinafter, we assume always the optimum saturation level for each value of m in the proposed quantization schemes, as referred previously.

Figures 4 and 5 show a CIR and CFR examples respectively, before and after P-UQ quantization for channel 2 using just 5 bits per QI component of each non-zero tap of CIR. The difference between the original CIR and the reconstructed CIR, after being quantized can be measured in Fig. 4, which results in similar CFR curves in frequency domain with a low overhead rate.

In Fig. 6 we can observe the performance of the multi-user MISO system using precoding at the BS to cancel interference, with both strategies presented: P-UQ and RVQ (where the all the samples of CFR are quantized and fed back). The number of quantization bits used is changing (b is set to 5, 6 and 7, which corresponds to the values for B of 20, 24

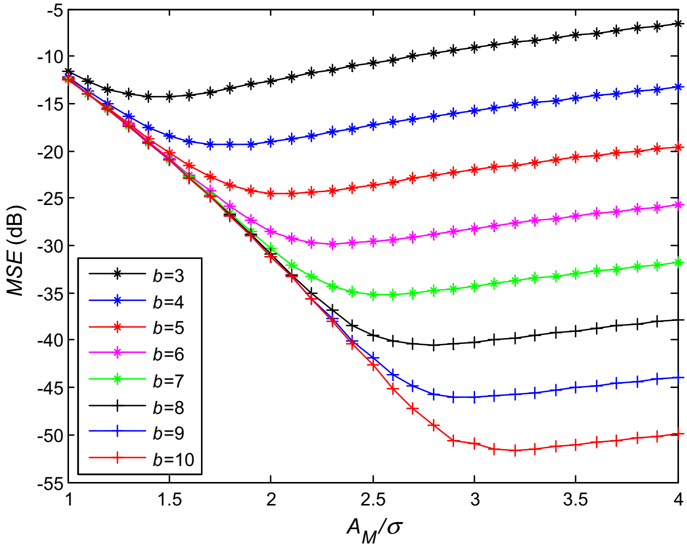

Fig. 2 MSE of CFR quantization in function of clipping value, for several number of quantization bits for channel 1

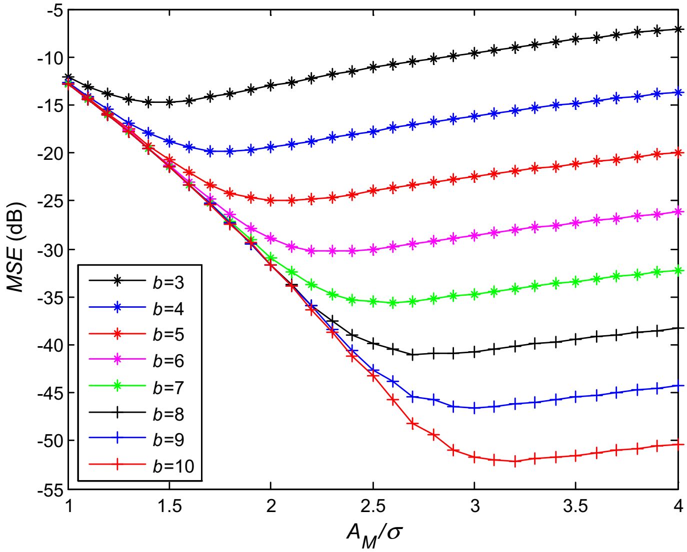

Fig. 3 MSE of CFR quantization in function of clipping value, for several number of quantization bits for channel 2

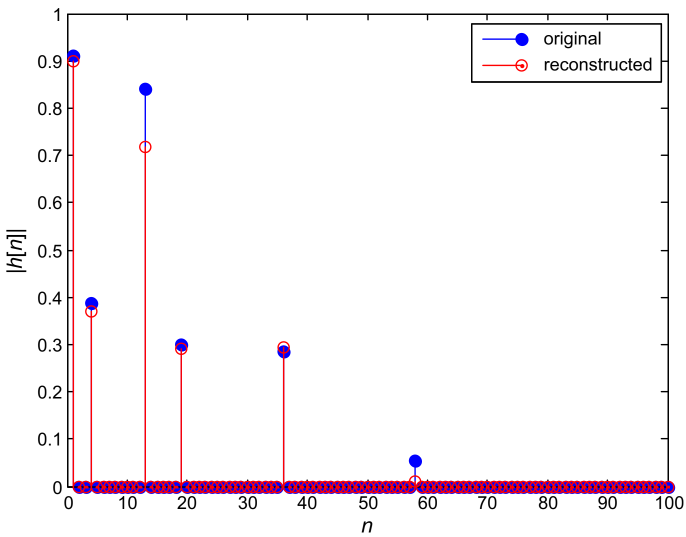

Fig. 4 CIR before and after UQ (original and recovered channels respectively in the legend) with $b = 5$ bits for channel 2

and 28, respectively so the feedback rate per user become the same). Despite the higher complexity and computational effort to perform the large codebooks, RVQ presents slightly better performances than P-UQ. The differences between both strategies become smaller as the number of quantization bits increases, since the curves tend to the perfect CSI feedback case.

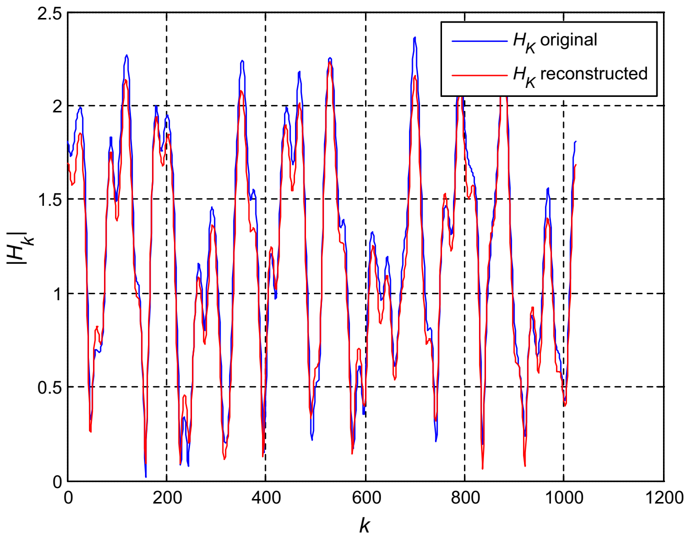

Fig. 5 CIR before and after UQ (original and recovered channels respectively in the legend) with b = 5 bits for channel 2

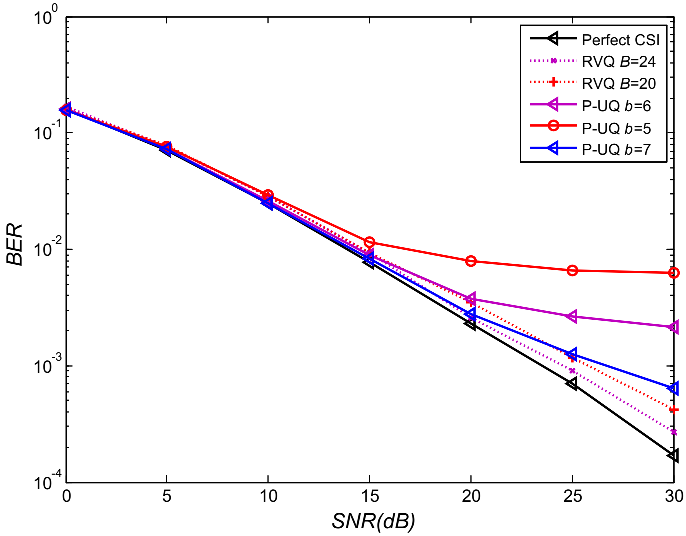

Fig. 6 BER performances using RVQ and UQ for channel 1 (quantization of all the samples of the CSI)

In Fig. 7 the results with channels 1 and 2 are compared, for the case of having $b = 5$ bits and $B = 20$ (as explained previously, this is an example that corresponds to the same number of quantization bits). We can observe some differences in having a uniform PDP channel and a multi-tap channel based on practical scenarios where taps are not equally spaced and have different mean powers. The behavior is the same, having just a slight degradation on the BER for the case of the pedestrian channel, referred as channel 2.

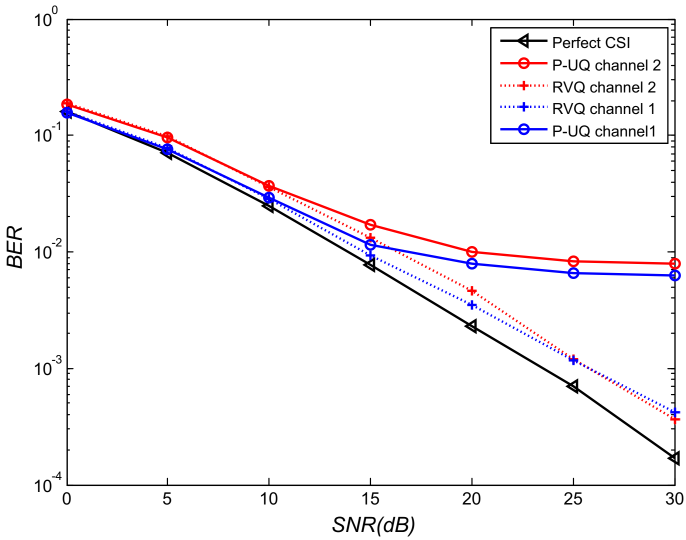

Fig. 7 BER performances using RVQ and UQ for channels 1 and 2, for $b = 5$ and $B = 20$ (quantization of all the samples of the CSI)

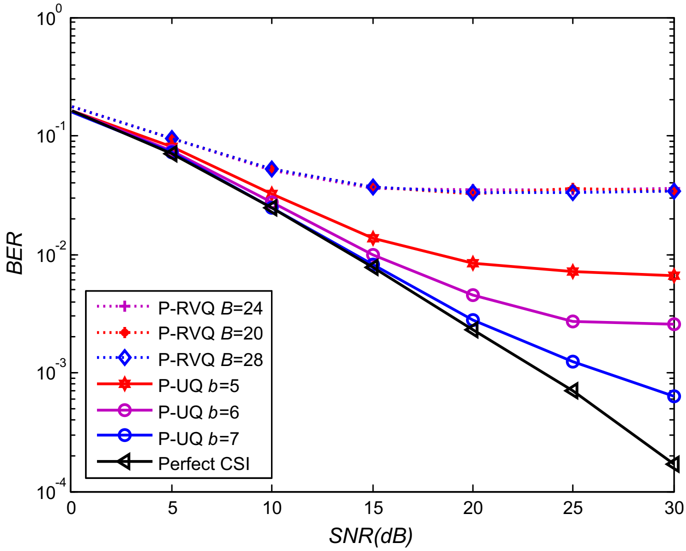

Fig. 8 BER performances using P-RVQ and UQ for CSI feedback for channel 1 (partial CFR samples quantization)

In Fig. 8 the results obtained for the proposed P-UQ strategy are presented and for comparison purposes, intending to have the same overhead conditions, we also consider quantization of just part of CFR with RVQ (P-RVQ). The performances with P-UQ always outperform the ones using P-RVQ, and the gains increase with the number of quantization bits. As the P-RVQ just quantizes the channel direction information, it is not possible to

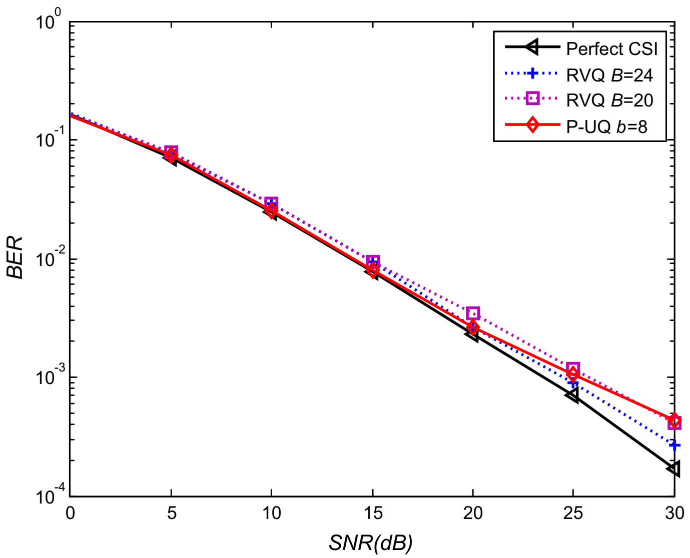

Fig. 9 BER performances using RVQ and UQ for channel 1 and with different number of quantization bits

recover the channel through part of the samples, independently of the quantization rate. Thus with RVQ we must quantize all the samples even if the channel is highly dispersive, contrarily to the UQ strategy.

Figure 9 plots the results for a scenario example where both strategies have similar performances. The performances of RVQ with quantization and feedback of all CFR samples with $B = 20$ has approximately the same performance as UQ with $b = 8$. Despite the same behavior, with UQ method we can reduce the number of quantization bits comparatively to RVQ, in a rate of $\frac{2bML}{BN}$, which is for this scenario of about 10% of the RVQ overhead for channel 1 and less than 1% for channel 2.

5 Conclusions

In this paper a UQ based quantization strategy is proposed and implemented, using a reduced amount of bits to feed back the CSIT in multi-user MIMO channels, exploiting channels' sparsity. A comparison is also made between the commonly used RVQ strategy with the proposed P-UQ one, for sparse channels. It was shown that we can have performances close to the ones with perfect CSI with a relatively low number of bits to quantize an appropriate number of samples of the channel. This method requires the quantization of only the non-zero CIR taps, reducing the amount of information to be fed back. We numerically showed that is not possible to similarly reduce the overhead for the RVQ without significant degradation. Thus, this work presents a significant contribution for next generation wireless networks that needs CSIT with reduced overheads.

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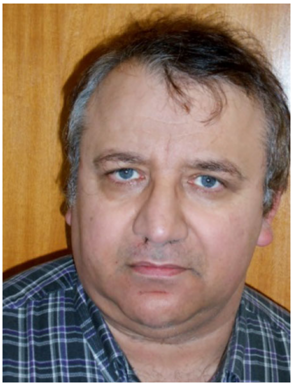

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