Self-Triggered Set-Valued Observers

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***Abstract*— This paper addresses the problem of high computational requirements in the implementation of Set-Valued Observers (SVOs), which places stringent constraints in terms of their use in applications where low computational power is available or the plant is sensitive to delay. It is firstly shown how to determine an overbound for the set-valued estimates, which reduces the overhead by limiting the number of inequalities defining those set-valued state estimates. In the particular setting of distributed gossip problems, the proposed algorithm is shown to have constant complexity. This algorithm is of prime importance to reduce the computational load and enable the use of such estimates for real-time applications. Results are also provided regarding the frequency of the triggers in the worst-case scenario. The performance of the proposed method is evaluated through simulation.**

I. Introduction

The problem of fault detection for general linear systems relates to that of determining, for any given set of inputs, noise and disturbances, if the measurements of the system can be produced by the model. An interesting instance of the problem is detecting faults in an asynchronous distributed environment, which refers to determining if any node enters an incoherent state given the observed measurements.

A special case of interest rises in the domain of randomized distributed algorithms, both due to their relevance in certain problems but also because of their unstructured nature, i.e., all nodes play the same role in the algorithm and the messages need not satisfy any particular type of time sequence. This class of algorithms is used for iterative solutions, because they are robust against packet drops and node failure. Applications of randomized algorithms range from selection and sorting [1] to consensus [2] and problems for which the solution requires a heavy computational burden. However, a set-valued estimate of the state is computed by union of the set of possible states generated by each transmission. By definition, the number of sets grows exponentially with the number of past time instants, i.e., the horizon N.

The study of Fault Detection and Isolation (FDI) problems has been a long standing research topic, since the early 70's (see [3]), but still poses remarkable challenges to both the scientific community and the industry (see, for example, the survey in [4] and references therein). Classical fault detection methods such as the ones proposed in [3], [5] and [6], rely on designing filters that generate residuals that should be *large* under faulty environments. Calculating thresholds for the residuals is typically cumbersome or poses stringent assumptions on the exogenous disturbances and measurement noise acting upon the system.

An alternative approach, based on Set-Valued Observers (SVOs) was described in [7]. The concept of SVOs was first introduced in [8] and [9], and further information can be found in [10], [11] and the references therein. The SVO-based solution alleviates the design complexity, while posing mild assumptions on the system. However, it also requires increased computational power when compared to classical FDI methodologies.

In the literature, another option is to use the concept of zonotopes, described in [12] and further developed in [13]. Those represent a different trade-off between intersections and unions of sets. One could also use the idea of interval analysis [14], although it introduces conservatism by not considering higher horizon values in their formulation, unlike the SVOs [7]. An alternative approach is adopted in this article, as described in the sequel.

The main contributions of this paper are as follows:

- given a specific structure for the matrix defining the polytope (i.e., the set-valued state estimate), it is shown how to compute an overbounding ellipsoid or ball;
- based on the concept of singular vectors, we show how a rotation can be found to prevent the approximation error of using boxes from going to infinity when the matrix defining the polytope is ill-conditioned;
- an algorithm is introduced that uses approximations to the *optimal* SVO estimates based on the previous methods, which is less computationally demanding, and self-triggers the computation of the aforementioned estimates only when necessary to ensure convergence;
- results are provided for the worst-case triggering frequency for a Linear Parameter-Varying (LPV) system;
- finally, for the special case of a distributed linear algorithm with a gossip property, it is shown that the overbounds are efficient to compute and propagate, since its complexity is constant.

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