Set-Consensus using Set-Valued Observers

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Abstract— **This paper addresses the problem of reaching consensus when the position measurements are corrupted by noise and taken at different time instants. Two different scenarios are considered, namely: when the sensor function is performed on a group of nodes and sent to those nodes at the same time; and a second case where the transmission is unicast but neighboring nodes receive the message due to the shared nature of the wireless medium. A solution involving Set-Valued Observers (SVOs) is proposed to maintain and update the set-valued estimates for the positions and how to drive the agents. The algorithm is proved to enforce convergence of the nodes to clusters up to a factor dependent on the uncertainty. The performance of the proposed algorithm is assessed through simulations, illustrating, in particular how the choice for selecting how to divide the area can reduce or increase the number of clusters.**

I. Introduction

The consensus problem is a distributed control task in which a set of agents have the objective of agreeing on a function of their initial state values by exchanging messages dictated by a given communication topology. Several multi-disciplinary applications of consensus algorithms have been reported in the literature. These include distributed optimization [1], [2]; motion coordination tasks, such as flocking, leader following [3], and rendezvous problems [4]; and resource allocation in computer networks [5].

Many contributions are available in the literature to solve consensus problems using linear distributed algorithms, in which each agent computes a weighted average between its state value and the state values of the agents to which it can communicate (see, e.g, [6], [7]). Many variations of this problem have been addressed in the literature considering, e.g., stochastic packet drops and link failures [8], [9], quantized data transmissions [10], and time-varying communication connectivity [6], event-triggered communications [11], and self-triggered communications [12].

Even though the problem of consensus is largely studied, most of the contributions assume the states of nodes to be exact values. In this article, we address the problem when two main issues are present: the measurements are performed with errors, which motivates the calculation of a set of possible "true state" realizations; and the measurements are performed at different time instants, which caters for the need to propagate the state using the dynamics of the system. In [13], the authors address the former issue, but specify a particular shape for the sets and assume all-to-all communication. By resorting to a different technique, we aim to relax these assumptions at the expenses of a higher computational cost.

We use the Set-Valued Observers (SVOs) framework, whose concept was introduced in [14] and [15] (further information can be found in [16] and [17] and the references therein) as a way to represent and propagate the set-valued state estimates. The approach allows us to easily consider any kind of linear dynamics for the agents, and also to incorporate disturbances and model uncertainties. Furthermore, it will allow us to use the predicted actions of each agent to better choose the control law to apply at each node based only on local information.

The main contributions of this paper are as follows:

- the use of SVOs to update the set representing the uncertainty about the position of the nodes;
- it is proposed that each node can estimate the future position of their neighbors from the information received in the shared medium;
- the positions of the nodes are shown to converge to the vicinity of the remaining nodes, with this vicinity being dependent on a measure of the uncertainty.

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