

# Computers, Modelling and Meaningful Learning in Science and Mathematics

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**RESUMO:** A computação científica é uma componente essencial para o desenvolvimento do conhecimento nas ciências e na matemática. No entanto, o equilíbrio existente na investigação entre computação, experimentação e teoria continua longe de estar incorporado nos correspondentes currículos do ensino secundário e universitário. Neste artigo discutimos o enquadramento teórico que suporta o desenvolvimento de currículos e ambientes de aprendizagem que integrem a modelação computacional e equilibrem as diferentes componentes da ciência e da matemática. Discutimos as vantagens da utilização do Modellus como elemento central deste programa e ilustramos com uma actividade de modelação computacional interactiva em física. Relatamos também os resultados da implementação deste programa em diversos cursos universitários.

**Palavras-chave:** Educação em ciências e matemática, física, modelação computacional, processos de cognição

**ABSTRACT:** Scientific computation is an essential component for the development of knowledge in science and mathematics. However, the balance that exists in research between computation, experimentation and theory is still far from being adequately incorporated in the corresponding high school and undergraduate university curricula. In this article we discuss the theoretical rationale supporting the development of curricula and learning environments that integrate computational modelling, while balancing the different knowledge components of science and mathematics. We discuss the advantages of using Modellus as a central element of such modelling approach and illustrate with an interactive computational modelling activity in physics. We also report the results of the implementation of this approach in several undergraduate university courses.

**Keywords:** Education in science and mathematics, computational modelling, physics, processes of cognition

## INTRODUCTION

In modern professional activities in the fields of science and mathematics there is no doubt that the corresponding epistemologies and cognition frames involve modelling processes that balance different elements of theory, experimentation and scientific computation. As a consequence, curricula and learning environments associated with science and mathematics education should be based on research inspired modelling cycle pedagogies, which aim to guide students through the different cognitive stages associated with the research processes involved in the development of knowledge and cognition in science and mathematics.

As a result of many research efforts (see, e.g., Blum, Galbraith, Henn & Niss, 2007; Handelsman et al., 2005; McDermott & Redish, 1999; Slooten, van den Berg & Ellermeijer, 2006), it has become increasingly clear that the learning processes are effectively enhanced when students are embedded in atmospheres with activities that approximately recreate the cognitive involvement of scientists in modelling research experiences. Contrary to the traditional instruction approaches which end up reducing the learning processes to a rote accumulation of facts or rules, these research inspired approaches have shown to be able to engage students in interactive and exploratory learning processes that are better suited to promote knowledge performance and to resolve cognitive conflicts with prior knowledge associated to common sense beliefs and incorrect scientific ideas.

Fundamental to the implementation of these modelling pedagogies is the early integration of activities with computational

knowledge and technologies, a goal that should be achieved in a way that reflects the interactive balance existing between the different epistemological components of science and mathematics (Ogborn, 1994).

In environments following research inspired methodologies, the introduction of computer modelling activities was initially focused on the use of programming languages, such as Fortran (Bork, 1967) and Pascal (Redish & Wilson, 1993) or, more recently Python (Chabay & Sherwood, 2008). This approach requires students to develop a working knowledge of programming, a fact that also happens when using scientific computation software such as Mathematica or Matlab. To reduce the cognitive opacity associated with programming notions and syntax, and focus the learning activities on the concepts of science and mathematics, several computer modelling systems were developed, for example the Dynamic Modelling System (Ogborn, 1985), Stella (High Performance Systems, 1997), Easy Java Simulations (Christian & Esquembre, 2007) and Modellus (Teodoro, 2002).

Below we discuss some important aspects of the theoretical rationale underlying this research inspired modelling approach and the advantages of using Modellus<sup>1</sup> as a central implementation system. As an illustrative example, we present an interactive computational modelling activity in physics. Finally we report the results of the implementation of this approach in several undergraduate university courses involving themes related to physics.

## **KNOWLEDGE, COGNITION AND LEARNING IN SCIENCE AND MATHEMATICS**

The development of knowledge and reasoning in science and mathematics involves cognitive processes that require rigorous declarative and procedural specifications of abstract concepts and of the connections existing between them. Crucial for the successful construction of models or theories is the interpretation and validation process which involves operational familiarization, stringent theoretical consistency requirements and a precise relation with the relevant referents, either in the universe of phenomena or in abstract mathematical worlds (Reif, 2008).

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<sup>1</sup> Modellus is a freely available software tool developed at FCT/UNL, see the webpage <http://modellus.fct.unl.pt>.

These characteristics make science and mathematics knowledge and reasoning distinctly different from the corresponding structures needed for common everyday actions. An important aspect of cognition that contributes to this level of difficulty is the need to distinguish between different but related concepts. Indeed, in science and mathematics there are many concepts to which correspond words that are frequently used in daily contexts, but whose meanings are changed both in essence and in degree of precision. Examples are, energy, field, force, function, to name just a few. When students try to adjust their prior knowledge to the new scientific contexts, the cognitive conflicts arising from the superficial similarity between elements of everyday knowledge and reasoning and elements of science and mathematics can then be a fertile ground for the development of persistent learning difficulties.

The history of science has many examples of analogous conceptual difficulties (see, e.g., Chalmers, 1999; Crump, 2001). The establishment of new concepts, models or theories and the substitution of old ones is a difficult cognitive process that involves progressive familiarization, or clarification of what is different and what is common, with the new structures of knowledge and reasoning. Simultaneously, there are also conceptual reification processes which lead to states of cognition where the new structures are manipulated as concrete and objective realities. Similarly, familiarization and reification are key cognitive aspects involved in the science and mathematics learning processes.

## **LEARNING IN SCIENCE AND MATHEMATICS: THE ROLE OF COMPUTERS**

The modelling processes of science and mathematics are strongly enhanced by the more powerful calculation, exploration, visualization and simulation capabilities associated with computational knowledge and technologies. Likewise, the expectation is that the processes of learning science and mathematics are effectively more meaningful with an ample use of computational modelling.

The matter of fact is that computers can be helpful cognitive artefacts that improve the fundamental familiarization and reification processes associated learning science and mathematics (Teodoro, 2005). Indeed, as

cognitive devices, computers are tools that allow the creation of learning environments in which the abstract conceptual entities of science and mathematics can be seen as concrete-abstract objects. Concrete in the sense that they can be manipulated on the screen and react as real objects and abstract because they can be scientific or mathematical constructs. For allowing this real time concrete interaction with the objects of science and mathematics, computers can act as powerful intellectual mirrors (Schwartz, 1989) for the cognitive activity of the mind, a role with the clear potential to enhance familiarization and reification and thus the process of meaningful learning.

Computers also open the possibility to introduce numerical analysis which can be conceptually simpler than analytical methods and allow the focus to be on meaning and semi-quantitative reasoning. With computers learning science and mathematics can follow processes of interactive modelling where multiple representations, such as graphs, tables and simulations, can be created and explored to make meaningful learning more effective.

#### **LEARNING SCIENCE AND MATHEMATICS: COMPUTERS, MODELLING AND MODELLUS**

To be able to fulfil their potential role in learning, computers should not be used to simply display text, images or simulations but as tools for modelling integrated in learning environments reflecting the exploratory and interactive nature of modern research in science and mathematics. In addition, the computational modelling process should be focused on the meaning of models and avoid learning opacity factors such as too much programming and specific software knowledge.

This educational challenge cannot be met by choosing a subset of programming languages and professional scientific computation software. It is necessary to develop computer software systems with computational modelling functionalities that contribute to a progressive growth of solid cognitive competencies in science and mathematics. In this context, Modellus stands out as a key computational modelling platform because it allows deeper familiarization and reification due to the following set of advantages: 1) An easy and intuitive creation of mathematical models using standard mathematical notation; 2) The

possibility to create animations with interactive objects that have mathematical properties expressed in the model; 3) The simultaneous exploration of multiple representations such as images, tables, graphs and animations; 4) The computation and display of mathematical quantities obtained from the analysis of images and graphs.

As a domain general environment for modelling, Modellus can be used to design learning activities which explorative and expressive modelling (Bliss & Ogborn, 1989; Schwartz, 2007). These modelling activities can be collaborative and conceived to address cognitive conflicts in the understanding of scientific and mathematical concepts, the manipulation of multiple representations of mathematical models and the interconnection between analytical and numerical approaches. They can also involve realistic problems to maximize the cognitive contact between models and real world referents. With Modellus and numerical methods the interactive exploration of models for more realistic problems can start at least in high school, allowing students a closer contact with the model referents, an essential cognitive element to appreciate the relevancy and power of models, necessarily a partial idealized representation of their referents.

#### **COMPUTATIONAL MODELLING WITH MODELLUS: AN EXAMPLE FROM PHYSICS**

Let us consider a computational modelling activity where students are challenged to perform a vertical jump interactively on the computer screen. This is an interesting problem involving fundamental concepts relevant for biomechanics and was one of the computational modelling series implemented in the biophysics course taken in 2009/2010 by first year biomedical engineering students at FCT/UNL. The starting point to construct a model of the jump is to acknowledge that the jumper can be represented by a point particle, located in the centre of mass, whose motion is governed by Newton's laws. Prior knowledge framing this problem involves knowledge obtained from observations of real jumps and knowledge about vectors and kinematics as well as about other examples of the application of the laws of classical dynamics.

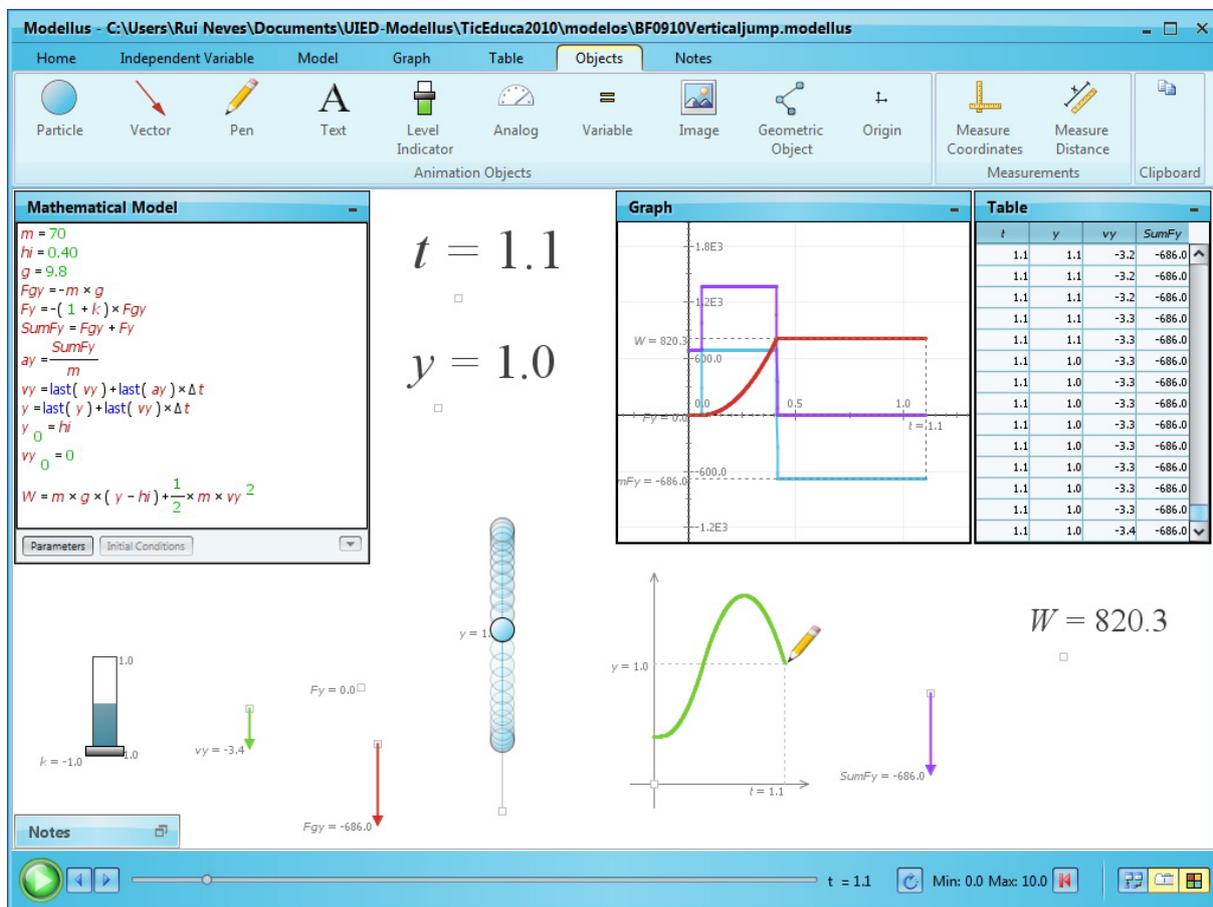
According to Newton's second law, the acceleration vector is obtained dividing the sum

of all the forces that act on the particle by the mass of the particle. If the force and the mass are known we can use Modellus to calculate the acceleration, the velocity and the position of a particle. Indeed, the acceleration and the velocity are, respectively, the vectors that measure the instantaneous rate of change of the velocity and of the position with time.

For the jumper's model Newton's equations of motion are written in the form of simple Euler iterations (see FIGURE I). A possible concrete setting is the following. Take the mass of the jumper equal to 70 kg and assume that in the initial upright position the centre of mass is 1 m above the ground. To

prepare the jump, the jumper bends his legs and lowers the centre of mass by 60 cm. Then assume the jumper applies a force on the ground that has an average magnitude equal to twice his weight. Assume further that this jump force acts during 0.3 s, the time interval needed raise the position of the centre of mass by 60 cm. The basic animation is constructed with a particle representing the jumper's centre of mass, vectors representing the forces applied on the jumper and a level indicator to control the magnitude of the force applied on the ground by the jumper (see FIGURE I). Several variable graphics and tables can also be displayed.

Figure I. The interactive vertical jump model with iterative Newton's equations. Relative to the initial upright position, the maximum jump height is 60 cm and the average work done to reach it is 820 J.



Because the magnitude of the jump force is an independent variable and the model is iterative, students can manipulate this vector at will and in real time perform the jump on the screen. To obtain a good simulation the students must choose an adequate numerical time step. This corresponds to the determination of an acceptable numerical solution of the equations of motion. While exploring the model, students

can determine, for example, the maximum height attained by the jumper and the average work done during the impulse for the jump. Students can change the model settings easily and analyse the jump physics for different jumpers and jump conditions. The possibility to change the mathematical model and immediately observe the consequences on the animation, graphs and tables is a powerful

cognitive element to enhance familiarization and reification. Students can also extend the model and perform, for example, a long jump.

## CONCLUSIONS: FIELD ACTIONS AND OUTLOOK

As part of the development of a computational modelling integration program started in 2008 at FCT/UNL, this and other computational modelling activities with Modellus have been implemented in the first year general physics (Neves, Silva & Teodoro, 2009, 2010; Teodoro & Neves, 2010) and biophysics courses of the biomedical engineering major. In all courses, the activities were successful in identifying and resolving several student difficulties in key physical and mathematical concepts. The possibility to have a real time visible correspondence between the animations with interactive objects and the object's mathematical properties defined in the model and the possibility to manipulate several different representations were instrumental to achieve this. As shown by the results of Likert scale based questionnaires, the majority of students reacted positively to the new component of the courses, showing clear preference for interactive and exploratory group work. Students considered Modellus helpful and user friendly in the processes of learning mathematical and physical models. The computational modelling activities with Modellus presented in digital PDF format with embedded video guidance and interactive space for answers were also considered to be interesting and well designed. Although the class implementation of the computational activities was successful, students manifested some caution and resistance to the novel approach, mainly because for them it meant extra work to master computational modelling besides just physics and mathematics. Students also felt that the content load was too heavy and that the time available for the computational modelling activities was insufficient, a problem felt even in the biophysics course where the time on the activities was doubled. Similar results were obtained with computational modelling activities with Modellus involving physics applied to introductory meteorology in a course gathering students from university majors in landscape architecture, environmental engineering, marine sciences and biology (Neves, Neves & Teodoro, 2009).

These important advances constitute definite improvements over the traditional instruction approaches, where the use of computers has been essentially limited to the presentation of text, images and simulations, or to a supporting role in data acquisition and analysis. However, a properly balanced integration of computational modelling with a clear measurable enhancement of cognition remains to be found. Important open questions left for future research are, for example: Is there an optimal set of tools that minimises cognitive opacity? If a course is organized into lectures, practical and laboratory work, what is the best way to integrate computational modelling? What is the most compelling and effective design for the interactive digital documents?

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