

Brane Dynamics, the Polytropic Gas and Conformal Bulk Fields

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Abstract

We consider in the Randall-Sundrum scenario the dynamics of a spherically symmetric 3-brane world when matter fields exist in the bulk. We determine exact 5-dimensional solutions which localize gravity near the brane and describe the dynamics of homogeneous polytropic matter on the brane. We show that these geometries are associated with a well defined conformal class of bulk matter fields. We analyze the effective polytropic dynamics on the brane identifying conditions which define it as singular or as globally regular.

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1 Introduction

In the Randall-Sundrum (RS) scenario [1, 2] the Universe we inhabit is a 3-brane world embedded in a Z_2 symmetric 5-dimensional anti-de Sitter (AdS) space. With

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two branes the scenario provides a solution to the hierarchy problem introducing an exponential warp in the fifth dimension [1]. Gravity is localized on the hidden positive tension brane and decays towards the visible brane which has negative tension. This leads to an exponential hierarchy between the weak and Planck scales and also to TeV scale Kaluza-Klein mass splittings and couplings [1]. A bulk scalar field may be used to stabilize the size of the fifth dimension [3, 4] and generate the Friedmann-Robertson-Walker (FRW) cosmology on the visible brane [5]. In addition it allows the low energy recovery of 4-dimensional Einstein gravity on the visible brane [6]. Two branes with positive tensions and an infinite fifth dimension may also be considered [7]. The exponential hierarchy is then obtained on the brane where the gravitational field is not localized and it turns out that for a low energy observer gravity is effectively described by 4-dimensional general relativity [7]. A warped infinite fifth dimension is also associated with a single positive tension brane to which the gravitational field is bound [2]. Again the low energy theory of gravity on the brane is effectively 4-dimensional general relativity [2, 8, 9] and the brane cosmology may be of the FRW type [10] (see also [11] where a single brane in a compact fifth dimension was discussed). The gravitational collapse of matter localized on a 3-brane was also addressed in this model [12]-[20]. To date realistic 5-dimensional collapse solutions remain to be found. The difficulty resides in achieving a simultaneous localization of gravity and matter which avoids the creation of naked singularities in the bulk. In this context it should be noted that exact solutions which may be interpreted as static black holes localized on a brane have been found for a 2-brane embedded in a 4-dimensional AdS bulk [13]. On the contrary, the exact 5-dimensional solutions discovered so far display singularities somewhere in the bulk [12, 14], [16]-[20].

In this letter we proceed with the investigation about the dynamics of a spherically symmetric RS 3-brane when conformal matter fields exist in the bulk [20]. We analyze the possibility of generating on the brane the dynamics of a perfect fluid with a polytropic equation of state. We dedicate special attention to its generalized Chap-

lygin phase which recently has been considered to be a viable alternative to model the accelerated expansion of the Universe [21]-[23].

2 Einstein Equations and Conformal Bulk Matter

Let us map the RS orbifold by a set of comoving coordinates (t, r, θ, ϕ, z) . The non-factorizable dynamical metric consistent with the Z_2 symmetry in z and with 4-dimensional spherical symmetry in (t, r, θ, ϕ) may be written as

$$d\tilde{s}_5^2 = \Omega^2 ds_5^2, \quad ds_5^2 = -e^{2A} dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2 + dz^2. \quad (1)$$

The functions $\Omega = \Omega(t, r, z)$, $A = A(t, r, z)$, $B = B(t, r, z)$ and $R = R(t, r, z)$ are Z_2 symmetric. Ω is the RS warp factor and R represents the physical radius of the 2-spheres. The classical dynamics is defined by

$$\tilde{G}_\mu^\nu = -\kappa_5^2 \left[\Lambda_B \delta_\mu^\nu + \frac{\lambda}{\sqrt{\tilde{g}_{55}}} \delta(z - z_0) (\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5) - \tilde{T}_\mu^\nu \right], \quad (2)$$

where Λ_B is the negative bulk cosmological constant, λ is the brane tension and $\kappa_5^2 = 8\pi/M_5^3$ with M_5 the fundamental 5-dimensional Planck mass. The brane is located at $z = z_0$ and the stress-energy tensor \tilde{T}_μ^ν is conserved in the bulk, $\tilde{\nabla}_\mu \tilde{T}_\nu^\mu = 0$.

Let us consider the special class of conformal bulk matter defined by $\tilde{T}_\mu^\nu = \Omega^{-2} T_\mu^\nu$ and assume that T_μ^ν depends only (t, r) . The matter represented by \tilde{T}_μ^ν is then not localized near the brane. Instead, because the conformal weight is -4 , the density and pressures increase with z and eventually become singular at the AdS horizon. The same problem is found in solutions with an extended black string singularity [12], [16, 18, 19]. The divergence of \tilde{T}_μ^ν makes it necessary to introduce two branes with opposite tensions in the bulk, which will intersect the space-time before the AdS horizon is reached. This was suggested in the first RS model [1] and the two branes will then have identical cosmological evolutions, though gravity will be localized on the first brane but not on the second. In a single brane model one must look for

solutions requiring a simultaneous localization of matter and gravity in the vicinity of the brane [19]. Assuming further that $A = A(t, r)$, $B = B(t, r)$, $R = R(t, r)$ and $\Omega = \Omega(z)$ we obtain [20]

$$G_a^b = \kappa_5^2 T_a^b, \quad \nabla_a T_b^a = 0, \quad G_z^z = \kappa_5^2 T_z^z, \quad (3)$$

and

$$6\Omega^{-2}(\partial_z \Omega)^2 = -\kappa_5^2 \Omega^2 \Lambda_B, \\ 3\Omega^{-1} \partial_z^2 \Omega = -\kappa_5^2 \Omega^2 \left[\Lambda_B + \lambda \Omega^{-1} \delta(z - z_0) \right]. \quad (4)$$

where the latin index represents the coordinates t, r, θ and ϕ . If the stress-energy tensor is $T_\mu^\nu = \text{diag}(-\rho, p_r, p_T, p_T, p_z)$, where ρ , p_r , p_T and p_z denote the bulk matter density and pressures, the non-trivial Einstein equations in Eq. (3) are given by

$$G_t^t = e^{-2B} \left(\frac{R'^2}{R^2} + 2\frac{R''}{R} - 2B'\frac{R'}{R} \right) - \frac{1}{R^2} - e^{-2A} \left(\frac{\dot{R}^2}{R^2} + 2\dot{B}\frac{\dot{R}}{R} \right) = -\kappa_5^2 \rho, \quad (5)$$

$$G_r^r = \frac{2e^{-2A}}{R} \left(\dot{R}' - A'\dot{R} - \dot{B}R' \right) = 0, \quad (6)$$

$$G_r^r = e^{-2B} \left(\frac{R'^2}{R^2} + 2A'\frac{R'}{R} \right) - \frac{1}{R^2} - e^{-2A} \left(\frac{\dot{R}^2}{R^2} + 2\frac{\ddot{R}}{R} - 2\dot{A}\frac{\dot{R}}{R} \right) = \kappa_5^2 p_r, \quad (7)$$

$$G_\theta^\theta = G_\phi^\phi = e^{-2B} \left(A'^2 + A'' - A'B' + A'\frac{R'}{R} + \frac{R''}{R} - B'\frac{R'}{R} \right) \\ - e^{-2A} \left(\dot{B}^2 + \ddot{B} - \dot{A}\dot{B} + \frac{\ddot{R}}{R} - \dot{A}\frac{\dot{R}}{R} + \dot{B}\frac{\dot{R}}{R} \right) = \kappa_5^2 p_T, \quad (8)$$

and

$$G_z^z = e^{-2B} \left(A'^2 + A'' - A'B' + \frac{R'^2}{R^2} + 2A'\frac{R'}{R} + 2\frac{R''}{R} - 2B'\frac{R'}{R} \right) - \frac{1}{R^2} \\ - e^{-2A} \left(\dot{B}^2 + \ddot{B} - \dot{A}\dot{B} + 2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - 2\dot{A}\frac{\dot{R}}{R} + 2\dot{B}\frac{\dot{R}}{R} \right) = \kappa_5^2 p_z. \quad (9)$$

where the dot and the prime denote, respectively, partial differentiation with respect to t and r . On the other hand the conservation equations are

$$\dot{\rho} + \dot{B}(\rho + p_r) + 2\frac{\dot{R}}{R}(\rho + p_T) = 0, \quad A'(\rho + p_r) + p_r' - 2\frac{R'}{R}(p_T - p_r) = 0 \quad (10)$$

and we also find [20] that p_z must satisfy a trace equation of state [11, 24]

$$\rho - p_r - 2p_T + 2p_z = 0, \quad (11)$$

for consistency.

For definiteness Ω may be taken to be $\Omega = \Omega_{\text{RS}} = l/(|z - z_0| + z_0)$ where $l = \sqrt{-6/(\Lambda_B \kappa_5^2)}$ and $\Lambda_B = -\kappa_5^2 \lambda^2/6$ [1, 2, 12]. This warp factor also holds in the two brane model as may be seen taking the periodicity and the Z_2 symmetry of the orbifold into account. Then Eq. (3) must be satisfied on both branes, in this sense twin universes with an identical cosmological evolution.

3 Polytropic Dynamics on the Brane

To behave as a polytropic fluid interacting with an effective cosmological constant on the brane, the diagonal conformal bulk matter should be defined by

$$\rho = \rho_{\text{P}} + \Lambda, \quad p_r + \eta \rho_{\text{P}}^\alpha + \Lambda = 0, \quad p_T = p_r, \quad p_z = -\frac{1}{2}(\rho_{\text{P}} + 3\eta \rho_{\text{P}}^\alpha) - 2\Lambda, \quad (12)$$

where ρ_{P} defines the polytropic energy density. Λ is the bulk quantity which mimics the brane cosmological constant. In the two brane model the usual 4-dimensional quantities are given by $\rho_4 = \rho_{\text{P}} L$, $\Lambda_4 = \Lambda L$ where $L = \kappa_5^2/\kappa_4^2$ is the length scale defining the interbrane distance and thus the size of the extra dimension. The parameters (α, η) characterize different polytropic phases.

Let us begin by solving the conservation equations in Eq. (10). Taking into account Eq. (12) we write them as follows

$$\dot{\rho}_{\text{P}} + \left(\dot{B} + 2\frac{\dot{R}}{R} \right) (\rho_{\text{P}} - \eta \rho_{\text{P}}^\alpha) = 0, \quad A' (\rho_{\text{P}} - \eta \rho_{\text{P}}^\alpha) - \eta \alpha \rho_{\text{P}}^{\alpha-1} \rho_{\text{P}}' = 0. \quad (13)$$

The contribution of the cosmological constant cancels out in Eqs. (13). Specializing to the case of a homogeneous energy density, $\rho_{\text{P}} = \rho_{\text{P}}(t)$, the metric function $A(t, r)$ can be safely set to zero. Combining with the off-diagonal Einstein equation (6), which

has solution $e^B = R'/H$ with $H = H(r)$ an arbitrary function of r , and separating the variables t and r we obtain [23]

$$\rho_P = \left(\eta + \frac{a}{S^{3-3\alpha}} \right)^{\frac{1}{1-\alpha}}, \quad (14)$$

where $\alpha \neq 1$, a is an integration constant and $S = S(t)$ is the Robertson-Walker scale factor of the brane world which is related to the physical radius by $R = rS$. For $\alpha = 1$ Eq. (14) is not defined. The appropriate limit is $\rho_P = b/S^{3-3\eta}$, where b is the corresponding integration constant and $\eta \neq 1$.

Next let us consider the diagonal Einstein equations in Eq. (3). First note that since $p_r = p_T$ we must have $G_r^r = G_\theta^\theta$, which is possible only if,

$$H^2 = 1 - kr^2, \quad (15)$$

where the constant k is the Robertson-Walker curvature parameter. Using Eqs. (5), (7), (8) and (12) we form the sum,

$$-G_t^t + G_r^r + 2G_\theta^\theta = -2\frac{\ddot{R}'}{R'} - 4\frac{\ddot{R}}{R} = \kappa_5^2 (\rho_P - 3\eta\rho_P^\alpha - 2\Lambda). \quad (16)$$

Then substituting $R = rS$ we obtain

$$\frac{\ddot{S}}{S} = -\frac{\kappa_5^2}{6} (\rho_P - 3\eta\rho_P^\alpha - 2\Lambda). \quad (17)$$

Applying the radial equation (7) then leads to

$$\dot{S}^2 = \frac{\kappa_5^2}{3} (\rho_P + \Lambda) S^2 - k. \quad (18)$$

Now, Eqs. (17) and (18) are linked by a derivative. They are consistent with each other when ρ_P obeys the conservation Eq. (13).

Finally, introducing Eqs. (17), (18) and (15) on Eq. (9) and using the expression for p_z given in Eq. (12) the zz component of the 5-dimensional Einstein equations given in Eq. (9) is seen to be an identity for all the parameters of the model. We have thus obtained the following 5-dimensional polytropic solutions for which gravity is confined near the brane

$$d\tilde{s}_5^2 = \Omega_{\text{RS}}^2 \left[-dt^2 + S^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right) + dz^2 \right], \quad (19)$$

where the brane scale factor S satisfies Eq. (18). It is easy to verify that if a 4-dimensional observer confined to the brane makes the same assumptions about the bulk degrees of freedom then she deduces exactly the same dynamics [20]. In fact, the non-zero components of the projected Weyl tensor [25] are given by

$$\mathcal{E}_t^t = \frac{\kappa_5^2}{4} (\rho_{\text{P}} - \eta \rho_{\text{P}}^\alpha), \quad \mathcal{E}_r^r = \mathcal{E}_\theta^\theta = \mathcal{E}_\phi^\phi = -\frac{\mathcal{E}_t^t}{3}. \quad (20)$$

As a consequence, the effective 4-dimensional dynamics is indeed defined by $G_t^t = -\kappa_5^2(\rho_{\text{P}} + \Lambda)$, $G_r^r = G_\theta^\theta = G_\phi^\phi = -\kappa_5^2(\eta \rho_{\text{P}}^\alpha + \Lambda)$. The 4-dimensional observer also sees gravity confined to the brane since she measures a negative tidal acceleration [26] given by $a_T = \kappa_5^2 \Lambda_B / 6$. Because of this the geodesics just outside the brane converge towards the brane and so for the 4-dimensional observer the conformal bulk matter is effectively trapped inside the brane.

At small S and for $\alpha < 1$ the polytropic dynamics is dominated by the homogeneous Oppenheimer-Snyder phase with $\rho_{\text{P}} = a^{1/(1-\alpha)} / S^3$ which corresponds to $\eta = 0$. If $\alpha \geq 1$ this is no longer true. For $\alpha = 1$ the dynamics is only that of dust if $\eta = 2/3$. For $\eta = -1/3$ we find radiation. If $\alpha > 1$ the small S dynamics is that of an effective cosmological constant. Note also that for $-1 \leq \alpha < 0$ and $0 < \alpha \leq 1$ there is an intermediate phase defined by the equation of state $p_{\text{P}} = -\alpha \rho_{\text{P}}$ which satisfies the dominant energy condition. If $|\alpha| > 1$ this condition is violated. For large S the dynamics is dominated by an effective cosmological constant term and it corresponds to $a = 0$ if $\alpha \neq 1$ or $b = 0$ if $\alpha = 1$.

In what follows we consider $-1 \leq \alpha < 0$. This is the generalized Chaplygin phase of the polytropic gas. To analyze it [27, 28] let us re-write Eq. (18) (with Eq. (14) substituted in) to define the potential $V(S)$ as follows

$$S\dot{S}^2 = V(S) = \frac{\kappa_5^2}{3} \left[\left(\eta S^{3-3\alpha} + a \right)^{\frac{1}{1-\alpha}} + \Lambda S^3 \right] - kS. \quad (21)$$

Naturally, the dynamics depends on Λ , k , η , a and α . Let us consider some special illustrative examples which can be treated analytically.

Starting from an initial state, a rebound will occur whenever $\dot{S} = 0$ and $S \neq 0$. This happens when $V(S) = 0$. To begin with consider $k = 0$. Then introducing $Y = S^{3-3\alpha}$ the potential V is given by

$$V = V(Y) = \frac{\kappa_5^2}{3} \left[(\eta Y + a)^{\frac{1}{1-\alpha}} + \Lambda Y^{\frac{1}{1-\alpha}} \right]. \quad (22)$$

Next take $\eta > 0$ and $a > 0$ [23]. If $\Lambda > 0$ then $V > 0$ for all $-1 \leq \alpha < 0$ and $S \geq 0$. Consequently, the Chaplygin shells may either expand continuously to infinity or collapse to the singular epoch at $S = 0$ where $V(0) = \kappa_5^2 a^{1/(1-\alpha)}/3 > 0$. Note that this is also what happens if $\Lambda = 0$. If $\Lambda < 0$ then two new possibilities arise. For $\eta \geq (|\Lambda|)^{1-\alpha}$ the Chaplygin gas is dominant and the potential is positive for all $Y \geq 0$. This implies an evolution of the type found for $\Lambda \geq 0$. If $\eta < (|\Lambda|)^{1-\alpha}$ then it is the cosmological constant which prevails. The allowed dynamical phase space is given by $0 \leq Y \leq Y_*$ where $Y_* = a/[(|\Lambda|)^{1-\alpha} - \eta]$ is the maximum regular rebound epoch. Since at $S = 0$ the shells meet the singularity these solutions are not globally regular on the brane.

It is important to note that the range $\eta < 0$ and $a < 0$ may be allowed in the presence of a non-zero cosmological constant. The same is not true for $\eta > 0$ and $a < 0$ or $\eta < 0$ and $a > 0$. Indeed, for these definitions the current experimental bounds are violated [23] and there is a singular epoch at $Y = -a/\eta > 0$ where V' diverges. If $\eta < 0$ and $a < 0$ the potential depends on $(-1)^{1/(1-\alpha)}$ and it is not defined for all values of $-1 \leq \alpha < 0$.

For $\alpha = -p/q$, $q > p$ with q and p , respectively, even and odd integers, the Chaplygin contribution to the potential V is positive for all $S \geq 0$ and so the evolution follows the case $\eta > 0$ and $a > 0$. However, if q is odd and p is even then the potential becomes

$$V = -\frac{\kappa_5^2}{3} \left[(|\eta|Y + |a|)^{\frac{q}{p+q}} - \Lambda Y^{\frac{q}{p+q}} \right] \quad (23)$$

and new dynamics develops. For a rebound, Λ must be positive and then the dynamical phase space is defined by $Y \geq |a|/((\Lambda)^{(p+q)/q} - |\eta|)$, $(\Lambda)^{(p+q)/q} > |\eta|$, where the lower limit of the interval is the only existing regular rebound point. In this phase the Chaplygin energy density is negative but the presence of the cosmological constant ensures that the total density ρ is not. Since $V(0) = -\kappa_5^2|a|^{q/(p+q)}/3 < 0$ the singularity at $S = 0$ is inside the forbidden region and does not form. These solutions are thus globally regular on the brane.

Another special case is that of $\Lambda = 0$. As we have seen for $k = 0$ there are only singular solutions without rebounding epochs. It is for $k \neq 0$ that new dynamics appears. With $Y = Z^3$ the potential V is given by

$$V = V(Z) = \frac{\kappa_5^2}{3} (\eta Z^3 + a)^{\frac{1}{1-\alpha}} - k Z^{\frac{1}{1-\alpha}}. \quad (24)$$

Consider $k > 0$, $\eta > 0$ and $a > 0$. The condition $V \geq 0$ is equivalent to $\mathcal{V} = \mathcal{V}(Z) \geq 0$ where

$$\mathcal{V} = \left(\frac{\kappa_5^2}{3} \right)^{1-\alpha} (\eta Z^3 + a) - k^{1-\alpha} Z. \quad (25)$$

Then there are no more than two regular rebound epochs in the allowed dynamical phase space. Since $\mathcal{V}(0) = (\kappa_5^2/3)^{1-\alpha} a > 0$ and $\mathcal{V}' = 3(\kappa_5^2/3)^{1-\alpha} \eta Z^2 - k^{1-\alpha}$, $\mathcal{V}'' = 6(\kappa_5^2/3)^{1-\alpha} \eta Z \geq 0$, this is controlled by the sign of \mathcal{V} at its minimum $\mathcal{V}_m = \mathcal{V}(Z_m)$ where $Z_m = \sqrt{(3k/\kappa_5^2)^{1-\alpha}/3\eta}$. If $\mathcal{V}_m > 0$ then there are no regular rebound points and the collapsing shells may fall from infinity to the singularity at $S = 0$ where $V(0) = \kappa_5^2 a^{1/(1-\alpha)}/3 > 0$. For $\mathcal{V}_m = 0$ we have just one regular fixed point $S = S_*$ which divides the phase space into two disconnected regions, a bounded region with the singularity at $S = 0$, $0 \leq S < S_*$, and an infinitely extended region, $S > S_*$, where the shells may expand increasingly faster to infinity. The solutions restricted to this region are regular. If $\mathcal{V}_m < 0$ then we have two regular rebound epochs $S = S_-$

and $S = S_+$ such that $S_- < S_+$. The region between them is forbidden as there the potential is negative. For $0 \leq S \leq S_-$ a shell may expand to a maximum radius rS_- and then rebound to collapse towards the singularity at $S = 0$. For $S \geq S_+$ the collapsing shells shrink to the minimum scale S_+ and then rebound to expand with ever increasing speed to infinity. For $\eta < 0$ and $a < 0$ we find the same type of dynamics but now only for the special values $\alpha = -p/q$, $q > p$ with q and p , respectively, even and odd integers.

If $k < 0$ then for $\eta > 0$ and $a > 0$ the potential is always positive and so there are only singular solutions without rebounding points. For $\eta < 0$ and $a < 0$ we must consider $\alpha = -p/q$, $q > p$ with q and p , respectively, odd and even integers to find solutions with rebounding points. The condition $V \geq 0$ is still equivalent to $\mathcal{V} \geq 0$ but now

$$\mathcal{V} = -\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} (|\eta|Z^3 + |a|) + |k|^{1-\alpha}Z. \quad (26)$$

Now since $\mathcal{V}(0) = -(\kappa_5^2/3)^{1-\alpha}|a| < 0$, $\mathcal{V}' = -3(\kappa_5^2/3)^{1-\alpha}|\eta|Z^2 + |k|^{1-\alpha}$ and $\mathcal{V}'' = -6(\kappa_5^2/3)^{1-\alpha}|\eta|Z \leq 0$, the sign of \mathcal{V} at its maximum $\mathcal{V}_M = \mathcal{V}(Z_M)$ where $Z_M = \sqrt{(3|k|/\kappa_5^2)^{1-\alpha}/3|\eta|}$ shows that the only possibilities are the existence of one or two regular rebound epochs. In the former case the classical brane stays forever in the fixed point and in the latter it oscillates back and forth between the two regular rebound epochs.

4 Conclusions

In this paper we have investigated if the localized gravitational dynamics of a polytropic gas could be generated in a RS brane world. To determine such solutions we have considered matter fields distributed in the AdS bulk and analyzed the 5-dimensional Einstein equations using a global conformal transformation whose factor is the Z_2 symmetric warp. Using bulk matter defined by a stress-energy tensor whose

conformal weight is -4 we have found 5-dimensional solutions for which gravity is localized near the brane and the dynamics of the conformal bulk matter on the brane is that of an homogeneous polytropic gas. We have analyzed the polytropic dynamics from the point of view of a 4-dimensional observer confined to the brane to find conditions for singular or globally regular behavior. For the examples analyzed we have found that the dynamical phase space displaying the evolution of polytropic shells on the brane has an overall pattern similar to that of homogeneous dark radiation [27]. As we have noted, since the conformal weight is -4 the bulk matter density and pressures increase with the coordinate of the fifth dimension. This is not a problem in a two brane model, but in a single brane model they diverge at the AdS horizon. Within the single brane model a solution to this problem requires the simultaneous localization of gravity and matter near the brane. An analysis of scenarios in which both matter and gravity are localized on a single brane will be published elsewhere.

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