Fault Detection for Cyber-Physical Systems: Smart Grid case

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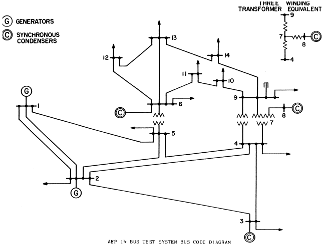
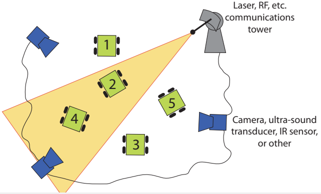
Outline

Motivation

- Sensor Smart Grids - Attacks or errors at the communication network can severely impact on the physical component.
- Robot Coordination - Formations of robots can also be seen as another example of a system with a communication network on top.





Cyber-physical System

- There are various physical components with their own dynamics.
- A communication network to manage the devices.
- We study the particular example of smart grids.
- Main issue: it is required a fast and distributed strategy to detect faults or attacks.

Smart grid model

- A smart grid is composed of:
  - n generator buses;
  - m load buses;
- network can be incorporated using its Laplacian matrix
- Using the dynamic linearized swing equation and the algebraic DC power flow equation, the model becomes:

$$N_c \dot{x}(t) = A_c x(t) + p(t) \tag{1}$$

- state $x = [\delta^\intercal \omega^\intercal \theta^\intercal]^\intercal \in \mathbb{R}^{2n+m}$ with:
  - $\delta \in \mathbb{R}^n$ - generator rotor angles;
  - $\omega \in \mathbb{R}^n$ - frequencies;
  - $\theta \in \mathbb{R}^m$ - bus voltage angles.

Problem Statement

- Given that the network is connected, $\theta(t)$ can be written using the other variables;
- The system is rewritten from an algebraic differential model to a kron-reduced version;
- After discretization, it becomes a Linear Time-Invariant (LTI) model:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ff(k) + Ed(k) \\ y(k) &= Cx(k) + Du(k) + Lf(k) + Nd(k) \end{aligned}, \tag{2}$$

Fault detection problem in Cyber-physical systems

How to perform fault detection without knowledge of the fault inputs? Is it a fast and distributed algorithm?

Centralized solution 1/2

- A node estimates the subnetwork of interest;
- No uncertainty in the model;
- New estimate for the state can be obtained by the inequality:

$$\underbrace{\begin{bmatrix} M(k)A^{-1} & -M(k)A^{-1}E \\ \bar{C} & 0 \\ 0 & \bar{I} \end{bmatrix}}_{M(k+1)} \begin{bmatrix} \mathbf{x} \\ \mathbf{d} \end{bmatrix} \leq \underbrace{\begin{bmatrix} m(k) + \tilde{u}(k) \\ \bar{y}(k+1) + \nu^{\star}1 \\ 1 \end{bmatrix}}_{m(k+1)} \quad (3)$$

- Propagation equation

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- Intersection with measurements

Centralized solution 1/2

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- Bounds on disturbances

Centralized solution 2/2

- The generalized solution exists for singular matrices A
- We can include previous time instants
- If we use a coprime factorization providing $P(z) = G^{-1}(z)Q(z)$ represented in

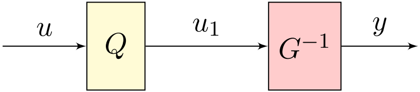


Figure: Schematic representation of the two coprime systems.

Decentralized solution

- Replace the known matrix A in the centralized version by:

$$A = A_0 + \sum_{\ell=1}^{n_\Delta} \Delta_\ell A_\ell \tag{4}$$

- Uncertainties parameters Δ_ℓ are used to represent the unknown dynamics
- The set can be obtained by computing the convex hull for each of the uncertainty vertex:

$$\tilde{X}(k+1) = \mathrm{co}\left(\bigcup_{\theta \in \mathcal{H}} \mathsf{Set}(\mathrm{M}_\theta(\mathrm{k}+1), \mathrm{m}_\theta(\mathrm{k}+1)) \right) \tag{5}$$

Results

- Centralized solution
  - If the system with n states is observable, convergence of the estimates is achieved in n time instants.
- Distributed solution
  - Convergence is governed by the slowest mode.
- In both cases, maximum magnitude for the attacker can be found by solving:

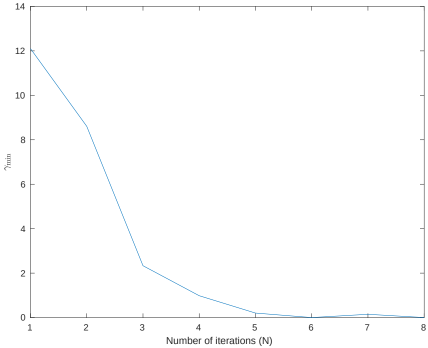
$$\gamma_{\text{min}} \geq \max_{A_H x \leq b_H} x^{\intercal} P_A x. \tag{6}$$

- P_A defining all the quadratic weights for the fault signals;
- A_H and b_H define the polytope containing all possible states.

Simulation Results (1/2)

Setup: Testbed network of 14 buses from IEEE.

- The average of the fault magnitude decreases with the number of past measurements.
- Attackers have a limited possibility to compromise the state without being detected.



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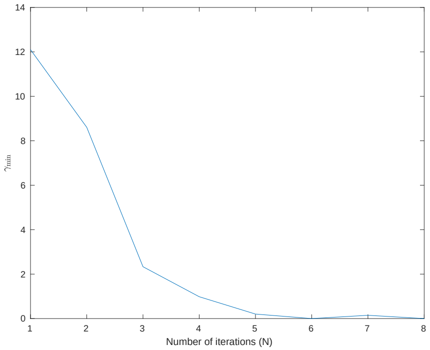
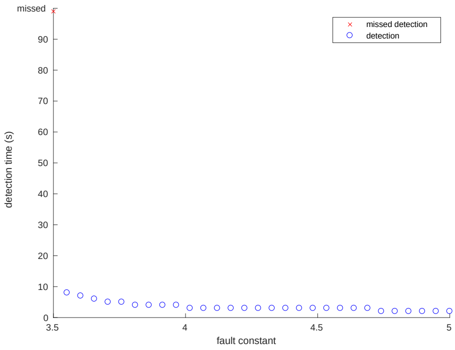


Simulation Results (2/2)

- The centralized solution detects faults of smaller magnitude.
- Detection was performed at most in n time instants.
- Detection for one of the observers in the network.
- Decentralized solution required a higher magnitude fault to ensure detection.



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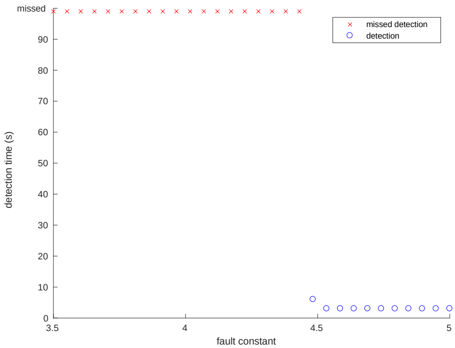

Concluding Remarks

Contributions:

- We have shown how to perform worst-case fault detection
    - centralized - one node with full knowledge of the network;
    - distributed - various node with a partial view.
- It is possible to give theoretical results about the convergence time;
- Finally, under the framework of distinguishability of models, it was possible to give worst-case bounds on the attacker signal.

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The end

- Thank you for your time.

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