Distributed Fault Detection Using Relative Information in Linear Multi-Agent Networks

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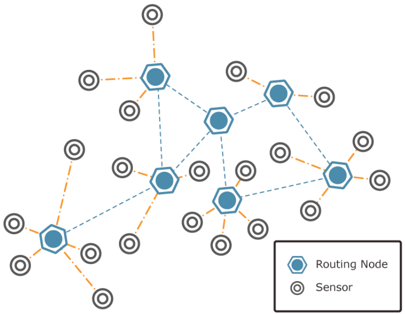
2015 IFAC SAFEPROCESS
Paris, France.

2nd September 2015

Outline

Motivation

- Networked Control Systems - Systems sharing a network and relative information.
- Distributed Fault Detection - Performing distributed detection may lead to unobservable but stable modes.



Set-valued estimates

- Fault detection using set estimators rely on testing if the set of possible states complying with the measurements is non-empty.
- Available relative measurements at every node.
- Distributed detection with nodes estimating only a partition of the states is a key requirement.
- Two main issues: unobservable modes might invalidate the design of observer-based fault detection methods and constructing set-valued estimates might mean a high computational load.

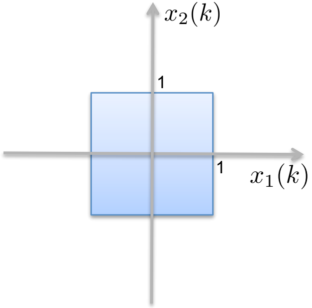
Motivating Example

- As an example consider the system:

$$x(k+1) = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d(k)$$

$$y(k) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(k) + n(k)$$

- $X(0) := \{v : \|v\|_\infty \leq 1\}$, $|n(k)| \leq 0.1$ and $|d(k)| \leq 1$, $\forall k > 0$.
- The set $X(1)$ becomes larger.
- The set-valued estimates are divergent invalidating a fault detection method based on set-valued estimators.



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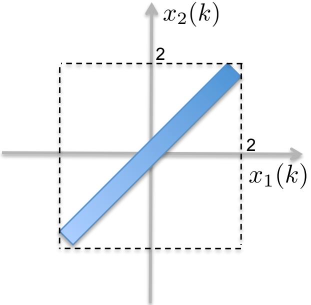


Problem Outline

- For a group of Linear Time-Invariant Systems (LTI) taking relative measurements, observability is lost.
- The sets $X(k)$ produced by a Set-Valued Observer (SVO) are increasing in size.

SVO-based Detection Problem for Unobservable Systems

How can we perform fault detection based on the set-valued estimates for the state, $X(k)$?

Problem Model

- Take an LTI of the form

$$S_i : \begin{cases} x_i(k+1) = Ax_i(k) + Bu_i(k) + Df_i(k) + Ed_i(k) \\ y_{ij}(k) = C(x_i(k) - x_j(k)), j \in \mathcal{J}_i \end{cases}$$

- $\mathcal{J}_i$ neighbor set of i
- The whole system can be written using the laplacian matrix $\mathcal{L}$

$$x(k+1) = \underbrace{(I_N \otimes A)}_{A_N} x(k) + \underbrace{(I_N \otimes B)}_{B_N} u(k) + \underbrace{(I_N \otimes D)}_{D_N} f(k) + \underbrace{(I_N \otimes E)}_{E_N} d(k)$$

$$y(k) = \underbrace{(\mathcal{L} \otimes C)}_{C_N} x(k)$$

Related Work

- Removing unobservable modes:

P. Menon and C. Edwards.
State transformation for the relative information case.
In *IEEE TAC*, 2014.

State transformation

$$T := T_s^{-1} \otimes I_n \ , \ T_s^{-1} := \begin{bmatrix} 1 & 0_{N-1}^{\intercal} \\ -1_{N-1} & I_{N-1} \end{bmatrix}.$$

Important step is to remove the first dimension since the relationship with the laplacian matrix

$$T_s^{\intercal} \mathcal{L} T_s = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{L}_r \end{bmatrix}$$

Proposed Solution

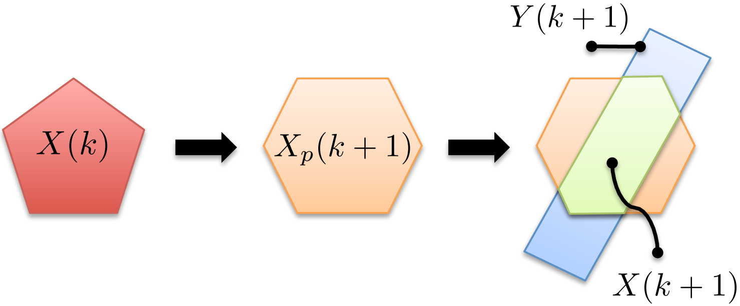
SVOs for the coprime factors of the system

- Obtain a left-coprime factorization of the system;
- Design a SVO for each factor;
- A fault is detected if the set-valued estimates of the two SVOs do not intersect.

Set-Valued Observers (SVOs)

Given the previous set $X(k)$:

- Using SVOs, the algorithm predicts $X_p(k+1)$ using the dynamics;
- Then, the set is intersected with the measurement set $Y(k+1)$.



Left-Coprime Factorization

- A left-coprime factorization produces two factors of the system G such that $G = N^{-1}M$.
- Factor N takes as input y and the noise and produces the output u_1.
- The SVOs will produce set-valued estimates for the internal states of M and N.
- A fault can be detected if there is no intersection between the output of M and N.

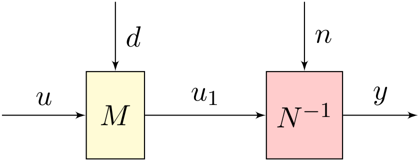


Figure: Schematic representation of the two coprime systems.

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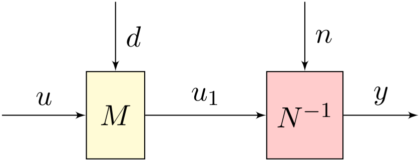

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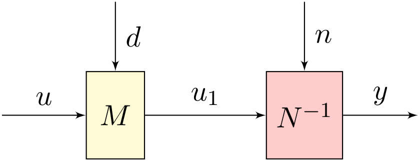


Figure: Schematic representation of the two coprime systems.

Main Result

Theorem 1

Consider a system G, where $x(k) \in \mathbb{R}^n$, which admits a left-coprime factorization such that $G = N^{-1}M$ and an SVO constructed for M and N providing estimates of u_1. Then:

i) *the set-valued estimates of u_1 have an infinite convergence rate, if G is observable;*

ii) *the set-valued estimates of u_1 convergence is governed by $\frac{1}{\lambda_{max}}$, where $\lambda_{max} := \max_{\lambda} |\lambda(A - KC)|$, if $\lambda_{max} < 1$.*

- For the unobservable case, convergence is governed by the slowest unobservable pole of $A - KC$.

Simulation Setup (1/2)

Setup: Each subsystem is a flexible link robot dynamical system of the form

$$\begin{bmatrix} \dot{\theta}^i_m \\ \dot{\omega}^{i_m} \\ \dot{\theta}^i_\ell \\ \dot{\omega}^{i_\ell} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_\ell}{J_m} & -\frac{B}{J_m} & \frac{K_\ell}{J_m} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_\ell}{J_\ell} & 0 & -\frac{L_\ell}{J_m} - \frac{mgh}{J_\ell} & 0 \end{bmatrix} \begin{bmatrix} \theta^i_m \\ \omega^{i_m} \\ \theta^i_\ell \\ \omega^{i_\ell} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_\tau}{J_m} \\ 0 \\ 0 \end{bmatrix} u^i$$

$$+ \begin{bmatrix} 0 \\ \frac{K_\tau}{J_m} \\ 0 \\ 0 \end{bmatrix} f^i + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{mgh}{J_\ell} \end{bmatrix} d^i, y_i = \sum_{j \in \mathcal{J}_i} C(x_i - x_j)$$

Simulation Setup (2/2)

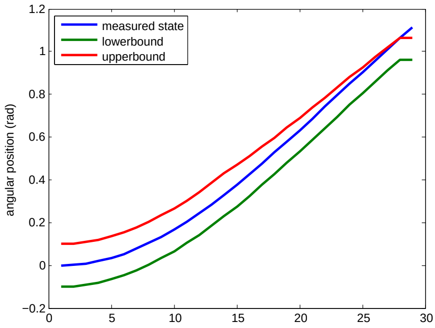
Where:

- $C = [I_3 \; 0_{3\times 1}]$;
- angular position θ_m^i and velocity of the motor shaft ω^{i_m};
- angular position θ_ℓ^i and velocity of the link ω^{i_ℓ};
- random 25-nodes network with a maximum degree of 3;
- sampling time of 0.01 seconds and simulations run for 100 discrete time steps.

Simulation Results (1/4)

Using the technique to remove the unobservable mode introduced in [Edwards:14] we can directly design a standard SVO.

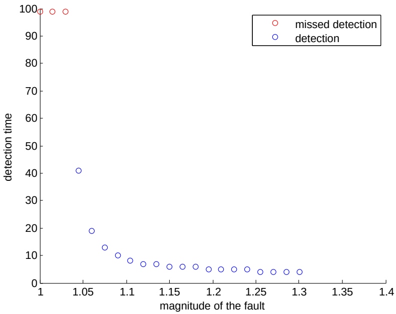
- When using the standard SVOs, the computed set represents the actual bounds for the admissible state values.
- A fault is detected when $X_p(k+1) \cap Y(k+1) = \emptyset$ which means that the true state $x(k+1) \notin X_p(k+1)$



Simulation Results (2/4)

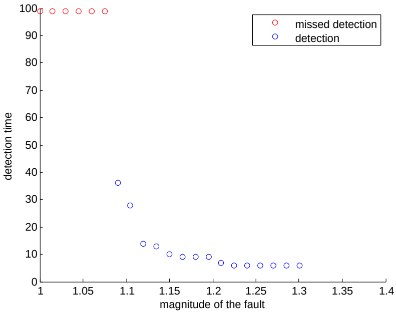
SVO designed for the coprime factorization

- First fault: constant signal mimicking an actuator fault $f(k) = B_N c$
- In a typical run, the proposed SVO is able to detect (blue circle) if the magnitude of the signal is greater than or equal to 1.05.
- Conclusion: Constant actuator faults are relatively easy to detect.



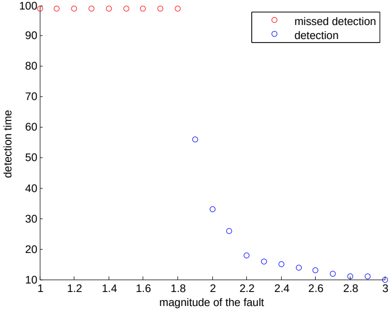
Simulation Results (3/4)

- Second fault: a random signal is added to the control input $f(k) = B_N r(k)$, which means that we can have time instants where it is close to having no fault.
- The proposed SVOs require a higher magnitude of around 1.1 to perform the detection.
- Conclusion: Successful detection of random actuator faults depends on the signal.



Simulation Results (4/4)

- Third fault: Unmodeled disturbances $f(k) = E_N r(k)$.
- The unmodeled disturbances affect less variables than the control input and for that reason required a higher magnitude bound to be detected.
- Conclusion: Signals that affect more variables are *easier* to detect.



Final Remarks

Contributions:

- The use of SVOs to compute the set-valued state estimates for the observable subsystem in a distributed fashion;
- The method for each node to estimate only their neighbors (distributed) or the whole system (centralized) is presented;
- For the case where the system is unobservable but detectable, convergence rate is shown to be a function of the slowest unobservable mode.

The end

- Thank you for your attention.

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