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Implications of anomalous gauge boson interactions to the fermion electromagnetic moments

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Abstract

We calculate the electromagnetic form-factors of the fermions induced by the anomalous gauge boson interactions contained in the operators $\mathcal{O}'_{DW}$ and $\mathcal{O}_{DB}$. The interplay between vertex corrections and gauge boson self-energies is studied in order to separate the non-universal form-factors. We apply the same procedure to reannalize previous results regarding other anomalous gauge boson interactions.

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1 Introduction

So far, the standard theory of electroweak interactions is consistent with all the available experimental results. The secret of this success seems to lie in its gauge symmetry structure, based on the SU(2)xU(1) group and served by a Higgs breaking mechanism into $U(1)_{QED}$. Gauge invariance assures renormalizability, a crucial condition to evaluate and predict higher order corrections, and implies another fundamental feature concerning the nature of gauge bosons self-interactions. Unlike the couplings to the matter fields, which require additional assumptions about the representations of matter (unless some other principle is postulated [1]), the couplings between gauge bosons are completely determined by local gauge invariance. However, no direct precise determination of the W, Z, γ self-couplings has been possible so far. The finding or elimination of anomalous boson self-interactions will reveal unknown high energy scale dynamics or confirm the very nature of the weak bosons, as that of the photon is already established. So, a great part of the significance of a machine like LEP2 lies in the possibility of directly measuring couplings such as WWZ and $WW\gamma$. That is well known and has been studied by many authors [2, 3, 4, 5, 6].

One may also look at indirect low-energy effects. A lot of work has been done focusing either on oblique corrections to 4-fermion amplitudes [7, 4, 8, 5] or on radiative corrections to the fermion-gauge boson couplings [9, 10, 11, 12, 13, 4]. Here, we are particularly interested in non-standard one-loop corrections to the electromagnetic moments of the fermions induced by anomalous $WW\gamma$ interactions. Previous works on that matter were based in one effective Lagrangian [3] that is the most general with up to 6-dimensional tri-linear operators if the gauge fields are restricted to be transverse ($\partial \cdot W = 0 = \partial \cdot Z$). That condition is satisfied if the gauge bosons are on-shell or coupled to massless fermions but is not necessarily true otherwise and therefore, there is a potential lack of generality in that mentioned effective Lagrangian. Indeed, we identified [14] a lot of independent couplings that vanish for transverse bosons but not for the spin zero degrees of freedom, the single constraint being electromagnetic gauge invariance.

Since however, the standard model gauge group has proven to be a good sym-

[1] It was showed in ref. [1] that the observed matter representations of the gauge groups SU(2) and SU(3) are the only ones that satisfy a Principle of Covariance with respect to basis transformations.

metry, even if in a hidden form, up to scales as high as M_Z, it is not so natural to expect non-invariant interactions at the scale of New Physics Λ which is itself above the very Fermi scale of weak symmetry breaking. Moreover, as emphasised by De Rújula *et. al.* [4], non- invariant operators produce divergent radiative corrections which, cut-off at the scale Λ, give rise to deviations from the predictions of the Standard Model such as the $M_W - M_Z - \mathrm{G}_\mu$ relation, which no longer decouple in the limit $\Lambda \to \infty$ even if originated from 6-dimensional operators suppressed by $1/\Lambda^2$. The anomalous interactions should then arise from an effective Lagrangian manifestly invariant under local SU(2)xU(1).

In the so-called linear realization, one includes also operators with the standard Higgs iso-doublet. A complete set of independent dimension-6 operators was identified by Buchmüller and Wyler [15]. Among the P and CP even operators, only seven generate gauge boson interactions: $\mathcal{O}_{\Phi,1}$, $\mathcal{O}_{BW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$ modify the 2-point Green functions at tree-level and $\mathcal{O}_{BW}, \mathcal{O}_{DW}, \mathcal{O}_{B}, \mathcal{O}_{W}, \mathcal{O}_{WWW}$, give rise to triple gauge boson couplings. In all cases except one, the $WW\gamma$ couplings reduce to the U(1)-invariant interactions usually cast [3] in a phenomenologic Lagrangian parametrized with Δk_γ and λ_γ (eq. (14) below). Their one-loop effects on the fermion electromagnetic couplings have been calculated. The exception is the operator $\mathcal{O}_{DW}$ as it yields interactions that are not electromagnetic gauge invariant by themselves but are rather undissociable from anomalous kinetic terms. That constitutes the primary motivation to study the one-loop corrections due to $\mathcal{O}_{DW}$ (in fact $\mathcal{O}'_{DW}$, a linear combination of $\mathcal{O}_{DW}$ and $\mathcal{O}_{WWW}$) and in particular to determine whether or not it produces fermion magnetic moments that could be used to set limits on the anomalous gauge boson interactions. We also do the calculations for the operator $\mathcal{O}_{DB}$ in view of its similarity with $\mathcal{O}_{DW}$.

In section 2 we derive the results of the one-loop Feynman diagrams. In section 3 we study the interplay between vertex corrections and gauge boson polarization functions including their longitudinal projections. It is then shown how to extract the non-universal electromagnetic couplings. The same procedure is used to reannalize and briefly overview in section 4 previous results on the electromagnetic form-factors induced by triple gauge boson interactions.

2 $\mathcal{O}'_{DW}$ and $\mathcal{O}_{DB}$ radiative effects

The effective Lagrangian is a linear combination of SU(2)xU(1) invariant operators,

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_i f_i \, \mathcal{O}_i \,, \tag{1}$$

that are functions of the Higgs covariant derivatives and gauge field strength tensors, denoted as (notation of refs. [5, 8]),

$$D_\mu \Phi = \left(\partial_\mu + i \frac{g'}{2} B_\mu + i \frac{g}{2} \sigma_i W^i_\mu \right) \Phi \,, \tag{2}$$

$$[D_\mu, D_\nu] = \hat{B}_{\mu\nu} + \hat{W}_{\mu\nu} = i \frac{g'}{2} B_{\mu\nu} + i \frac{g}{2} \sigma_i W^i_{\mu\nu} \,. \tag{3}$$

The operators $\mathcal{O}_{DW}$ and $\mathcal{O}_{DB}$ are defined as

$$\mathcal{O}_{DW} = \text{Tr} \left\{ \left[D_\mu, \hat{W}_{\nu\alpha} \right] \left[D^\mu, \hat{W}^{\nu\alpha} \right] \right\} \,, \tag{4}$$

$$\mathcal{O}_{DB} = -\frac{1}{2} g'^2 (\partial_\mu B_{\nu\alpha})(\partial^\mu B^{\nu\alpha}) \,. \tag{5}$$

In addition to quadratic terms, $\mathcal{O}_{DW}$ includes also trilinear couplings, but part of them are already present in $\mathcal{O}_{WWW} = \text{Tr} \left\{ \hat{W}_{\alpha\beta} \hat{W}^{\beta\gamma} \hat{W}_\gamma^\alpha \right\}$ as follows from the identity

$$\mathcal{O}_{DW} = -4 \, \mathcal{O}_{WWW} + 2 \, \mathcal{O}'_{DW} \,, \tag{6}$$

$$\mathcal{O}'_{DW} = \text{Tr} \left\{ \left[D_\mu, \hat{W}^{\mu\alpha} \right] \left[D^\nu, \hat{W}_{\nu\alpha} \right] \right\} \,. \tag{7}$$

Given the fact that $\mathcal{O}'_{DW}$ not only contains all the quadratic terms of $\mathcal{O}_{DW}$ and, just like $\mathcal{O}_{DB}$ and unlike $\mathcal{O}_{DW}$, is directly related through the equations of motion to the matter currents,

$$\mathcal{O}'_{DW} = -\frac{1}{2} g^2 J^a_T \cdot J^a_T \,, \tag{8}$$

$$\mathcal{O}_{DB} = -g'^2 J_Y \cdot J_Y \,, \tag{9}$$

we choose to replace $\mathcal{O}_{DW}$ by $\mathcal{O}'_{DW}$ in the set of linearly independent operators. The relation between the coupling constants in this basis ($\mathcal{O}'_{DW}$, $\mathcal{O}_{WWW}$) and in the basis ($\mathcal{O}_{DW}$, $\mathcal{O}_{WWW}$) is straightforward: $f'_{DW} = 2 \, f_{DW}$, $f'_{WWW} = f_{WWW} - 4 \, f_{DW}$.

$\mathcal{O}_{DB}$ and $\mathcal{O}'_{DW}$ generate the following two-point Green functions ($f'_{DB} = 2 \, f_{DB}$):

$$i \, \Pi^{\mu\nu}_{*+-} = -i \frac{f'_{DW} \, g^2}{\Lambda^2} \, p^2 \left(p^2 g^{\mu\nu} - p^\mu p^\nu \right) \tag{10}$$

for the charged Ws and

$$i\,\Pi^{\mu\nu}_{*ab} = -i\left[\frac{f'_{DW}\,g^2}{\Lambda^2}\,R^3_a R^3_b + \frac{f'_{DB}\,g'^2}{\Lambda^2}\,R^B_a R^B_b\right] p^2\left(p^2 g^{\mu\nu} - p^\mu p^\nu\right)\,, \tag{11}$$

for the neutral physical particles γ and Z; R is the weak rotation matrix from γ, Z to the W^3, B weak basis. The Feynman rule for the anomalous coupling of $W^-_\alpha W^+_\beta W^3_\mu$ with incoming momenta respectively $x, y, -k$, is

$$\begin{aligned} -\,i\,g\,\Gamma^{\alpha\beta\mu}_{*} = -i\,g\frac{f'_{DW}\,g^2}{\Lambda^2}\quad & \Big\{\left(x^2\delta^\alpha_\rho - x^\alpha x_\rho\right)\Gamma^{\rho\beta\mu}_0 + \left(y^2\delta^\beta_\rho - y^\beta y_\rho\right)\Gamma^{\alpha\rho\mu}_0 + \\ & + \left(k^2\delta^\mu_\rho - k^\mu k_\rho\right)\Gamma^{\alpha\beta\rho}_0\Big\}\,, \end{aligned} \tag{12}$$

$$\Gamma^{\alpha\beta\mu}_0 = \left(x^\mu - y^\mu\right) g^{\alpha\beta} - \left(x^\beta + k^\beta\right) g^{\alpha\mu} + \left(y^\alpha + k^\alpha\right) g^{\beta\mu}\,. \tag{13}$$

For special processes like W pair production at LEP2 this interaction is not independent from the ones that have been considered so far [3, 5, 6] assembled in the phenomenologic Lagrangian

$$\mathcal{L}_{WW\gamma} = -i\,e\,\Delta k_\gamma\, W^+_\mu W_\nu F^{\mu\nu} - i\,e\frac{\lambda_\gamma}{M_W^2} W^+_{\lambda\mu} W^{\mu\nu} F^\lambda_\nu\,. \tag{14}$$

Indeed, it reduces for transverse bosons ($\partial \cdot W^a = 0$) to a simple form-factor of the standard model vertex:

$$\Gamma^{\alpha\beta\mu}_* \longrightarrow \frac{f'_{DW}\,g^2}{\Lambda^2}(x^2 + y^2 + k^2)\Gamma^{\alpha\beta\mu}_0\,. \tag{15}$$

However, when considering one-loop effects neither the running with the momenta nor the longitudinal degrees of freedom can be discarded in advance. In addition, as will be seen below, that interaction is not electromagnetic gauge invariant per se (unlike the ones of eq. (14)) but only when associated with certain W kinetic terms. This was the primary motivation to study its one-loop effects.

The standard model $WW\gamma$ coupling is just $-ie\Gamma_0$ in the R_ξ gauge but we used the Fujikawa gauge-fixing condition [16] for the W field namely,

$$\mathcal{L}_W[\mathrm{gf}] = -1/\xi_W\left|\,\partial^\mu W^+_\mu + i\,eA^\mu W^+_\mu + i\,gv\,\xi_W\,\phi^+/2\,\right|^2\,. \tag{16}$$

It eliminates the trilinear coupling of the photon with the W and Goldstone boson ϕ^+ and in addition, shifts the standard model $WW\gamma$ coupling to:

$$\Gamma^{\alpha\beta\mu}_{\mathrm{SM}} = \Gamma^{\alpha\beta\mu}_0 + 1/\xi_W\left(x^\alpha g^{\beta\mu} - y^\beta g^{\alpha\mu}\right)\,. \tag{17}$$

The advantage of such W covariant gauge lies in that this $WW\gamma$ coupling and the W full propagator obey a Ward identity:

$$-iek_\mu \Gamma^{\alpha\beta\mu}_{\rm SM} = e\left(G^{-1}(y) - G^{-1}(x)\right)^{\alpha\beta} \ , \tag{18}$$

$$G^{\alpha\beta} = \frac{i}{p^2 - M_W^2}\left[-g^{\alpha\beta} + (1-\xi_W)\frac{p^\alpha p^\beta}{p^2 - \xi_W M_W^2}\right] . \tag{19}$$

Then, since the $\mathcal{O}'_{DW}$ coupling and self-energy satisfy a similar identity,

$$-\,iek_\mu \Gamma^{\alpha\beta\mu}_{*} = e\left(-i\Pi_*(y) + i\Pi_*(x)\right)^{\alpha\beta}_{+-} \ , \tag{20}$$

one obtains an automatic Ward-Takahashi identity for the one-loop fermion vertex. Note that the last equation just proves the point that the above anomalous $WW\gamma$ interaction is not electromagnetic gauge invariant by itself and therefore, cannot be reduced to the couplings of the phenomenologic Lagrangian of eq. (14).

We calculated the one-loop corrections to the $f\bar{f}\gamma$ vertices (once for all denoted as $-i\Delta\Gamma^\mu_\gamma$) using dimensional regularization. There are two kinds of diagrams: the ones where the photon couples to the fermion line and the internal gauge bosons carry anomalous self-energies and the ones where the photon couples to the charged W (only in the case of $\mathcal{O}'_{DW}$) either with anomalous $WW\gamma$ coupling or with the standard one plus anomalous W self-energy. Keeping only the divergent terms, we obtained for a on-shell fermion with charge Q and isospin

$$T_3 = t_3(1-\gamma_5)/2\,, \qquad t_3 = \pm 1/2\,, \tag{21}$$

the following results after fermion wave function renormalization:

$$-i\Delta\Gamma^\mu_\gamma = -e\,\frac{g^2}{2}\,\frac{f'_{DW}\,g^2}{\Lambda^2}\,J\left(k^2\gamma^\mu - k^\mu \not{k}\right)\left[Q\frac{1-\gamma_5}{2} - T_3 + 3(1-\xi_W)T_3\right], \tag{22}$$

$$-i\Delta\Gamma^\mu_\gamma = -e\,g'^2\,\frac{f'_{DB}\,g'^2}{\Lambda^2}\,J\left(k^2\gamma^\mu - k^\mu \not{k}\right)\frac{2}{3}Q(Q-T_3)^2\,. \tag{23}$$

Here, J is the integral in momentum space

$$J = \int \frac{d^dp}{(2\pi)^d}\frac{1}{\left(p^2 - M_W^2\right)^2} = \frac{i}{16\pi^2}\ln\frac{\Lambda^2}{M_W^2} \ , \tag{24}$$

where the right-hand side is the result of a cut-off regularization, the cut-off scale Λ naturally identified with the scale of New Physics. The ξ_W dependence of the results is not surprising given the interplay between vertex and vacuum polarization functions already present in the standard model.

3 The physical electromagnetic couplings

It is well known that in non-abelian theories the radiative corrections to the fermion-gauge boson couplings are not gauge-invariant per si neither are the gauge boson self-energies. Gauge invariant quantities are obtained as certain combinations of vacuum polarization and universal vertex functions. In the case of the standard model, some of the non-trivial features can be reduced to simple corrections of g and g' propagated to all gauge boson couplings and masses [17]; when considering anomalous gauge boson couplings one also finds additional mixing between W^3 and B in the fermion vertices [8]. We present here an extension suitable for massive fermions covering the interplay between the longitudinal parts of the boson propagators and the anapole type of vertex.

In a 4-fermion amplitude at one-loop level, there are vacuum polarization diagrams and corrections to the vertices, the latter denoted as $-i\Delta\Gamma_a^\mu$ for each boson $a = \gamma, Z, W^\pm$. It is convenient to separate the gauge boson propagators and self-energies in their transverse and longitudinal components:

$$G_a^{\mu\nu} = -i\left[\frac{P_T^{\mu\nu}}{k^2 - M_a^2} + \frac{P_L^{\mu\nu}}{k^2/\xi_a - M_a^2}\right] , \tag{25}$$

$$i\Pi_{ab}^{\mu\nu} = -i\left(P_T^{\mu\nu}\pi_{ab} + P_L^{\mu\nu}\rho_{ab}\right) , \qquad a, b = \gamma, Z, W^\pm \tag{26}$$

where

$$P_T^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} = g^{\mu\nu} - k^\mu k^\nu/k^2 . \tag{27}$$

By looking at the dependence of the amplitudes on the fermion quantum numbers, one comes to the conclusion that a certain kind of one-loop vertices give the same results as the vacuum polarization diagrams. In the case of neutral currents, the most general form of these universal couplings is:

$$\delta\Gamma_a^\mu\,[\text{univ}] = \gamma^\mu\left(q\,\Lambda_{\gamma a} + q_Z\,\Lambda_{Za}\right) - k^\mu \not{k}\left(q\,A_{\gamma a} + q_Z\,A_{Za}\right) , \quad a = \gamma, Z \tag{28}$$

where Λ_{ab}, A_{ab} are flavour independent functions of k^2 and $q = Qe$, q_Z are the electric and Z^0 fermion charges:

$$q_Z = \left(T_3 - Q\sin^2\theta_W\right)\sqrt{g^2 + g'^2} . \tag{29}$$

Of course, one could write the universal couplings in terms of Q and T_3 as well, but the above formulation makes it particularly easy to show that the sum of vertex and

vacuum polarization diagrams remains invariant if one replaces the self-energies and couplings with:

$$\pi'_{ab} = \pi_{ab} - (k^2 - M_a^2)\Lambda_{ab} - (k^2 - M_b^2)\Lambda_{ba}\,, \tag{30}$$

$$\rho'_{ab} = \rho_{ab} - (k^2/\xi_a - M_a^2)(\Lambda_{ab} - k^2 A_{ab}) - (k^2/\xi_b - M_b^2)(\Lambda_{ba} - k^2 A_{ba}), \tag{31}$$

$$\Delta\Gamma'^{\mu}_{\ a} = \Delta\Gamma^{\mu}_{a} - \gamma^\mu\,(q\,\Lambda_{\gamma a} + q_Z\,\Lambda_{Za}) - k^\mu \not{k}\,(q\,A_{\gamma a} + q_Z\,A_{Za})\;. \tag{32}$$

In this way one can obtain gauge-invariant quantities[2].

We define the electromagnetic field as the one that couples universally with the electric charge exclusively and obeys the Maxwell equations of motion which implies a zero photon mass and a dynamical decoupling from the Z^0 field. The first condition is realized by the cancellation of the universal part of $\Delta\Gamma^\mu_\gamma$ proportional to q_Z by the $\Lambda_{Z\gamma}$ and $A_{Z\gamma}$ terms. Hence, the remaining universal component of the electromagnetic coupling reduces to a running electric charge unit $e(k^2)$:

$$\Delta\Gamma'^{\mu}_{\ \gamma}\,[\text{univ}] = Q\,\Delta e(k^2)\,\gamma^\mu\,. \tag{33}$$

The second condition is realized by chosing the functions Λ_{ab} so as to annihilate the real part of the renormalized polarization functions:

$$\Re \begin{pmatrix} \pi'_{\gamma\gamma}(k^2) & \pi'_{\gamma Z}(k^2) \\ \pi'_{\gamma Z}(k^2) & \pi'_{ZZ}(k^2) - \pi'_{ZZ}(M_Z^2) \end{pmatrix} \equiv 0\,. \tag{34}$$

As a result, the renormalized transverse inverse propagator is just given by

$$\begin{pmatrix} k^2 + \Im\{\pi_{\gamma\gamma}\} & \Im\{\pi_{\gamma Z}\} \\ \Im\{\pi_{\gamma Z}\} & k^2 - M_Z^2 + \Im\{\pi_{ZZ}\} \end{pmatrix}, \tag{35}$$

where $\Im$ stands for the imaginary part: the $\gamma - Z$ decoupling is manifest. Finally, $A_{\gamma\gamma}$, $A_{\gamma Z}$ and the longitudinal polarization functions $\rho_{\gamma\gamma}$, $\rho_{\gamma Z}$ are immaterial for on-shell fermion amplitudes, as they give contributions proportional to the operator $q\,k^\mu \not{k}$ that vanishes in that case.

[2]Eq. (30) is in agreement with ref. [8] where the same relations were derived for particular Λ functions.

The solution of the above set of conditions is then:

$$\Lambda_{\gamma\gamma} = \Re\left\{\pi_{\gamma\gamma}(k^2)\right\}/2k^2\,, \tag{36}$$

$$\Lambda_{\gamma Z} = \Re\left\{\pi_{\gamma Z}(k^2) - (k^2 - M_Z^2)\Lambda_{Z\gamma}\right\}/k^2\,, \tag{37}$$

$$\Lambda_{ZZ} = \Re\left\{\pi_{ZZ}(k^2) - \pi_{ZZ}(M_Z^2)\right\}/2(k^2 - M_Z^2)\,. \tag{38}$$

When these expressions are substituted in eq. (32) one obtains for the neutral current amplitudes what is called improved Born approximation [17, 18]: it means that the sum of the amplitudes with boson self-energies and vertex radiative corrections is written with expressions where the gauge boson propagators are the free particle propagators (except for imaginary parts, see eq. (35)) and the radiative corrections are collected in the new *gauge-invariant* vertices given by eq. (32). They contain both flavour dependent and universal form-factors. The latter, once added to the tree-level vertices, can be expressed as running coupling constants. The boson self-energies contribute only to the universal vertices as follows: $\Lambda_{\gamma\gamma}$ contributes to the running electric charge unit and $\Lambda_{ZZ}, \Lambda_{\gamma Z}$ to the running coefficients of T_3 and Q, in the Z^0 coupling (eq. (29)).

In what regards the effects induced by $\mathcal{O}'_{DW}$ and $\mathcal{O}_{DB}$ it is not difficult to isolate from eqs. (22, 23) the flavour dependent electromagnetic coupling as

$$\Delta\Gamma^\mu_\gamma[f] = Q\left(k^2\gamma^\mu - k^\mu \not{k}\right)\left[-\beta\,\frac{f'_{DW}\,g^2}{\Lambda^2}\,\gamma_5 + \frac{8}{3}\beta'\,\frac{f'_{DB}\,g'^2}{\Lambda^2}(Q - T_3)^2\right]\ln\frac{\Lambda^2}{M_W^2}\,, \tag{39}$$

where

$$\beta = \frac{g^2 e}{64\pi^2}\,, \qquad \beta' = \frac{g'^2 e}{64\pi^2}\,. \tag{40}$$

This expression is clearly gauge independent and does not receive contributions from the boson self-energies. The remaining terms can be put in the form of eq. (28) and have to be summed to the universal contributions arising from the boson self-energies as explained before. As far as the electromagnetic interaction is concerned the overall result is a running $\alpha_{\rm QED}$ but such kind of contribution already appears in the tree-level self-energy specified by eq. (11) yielding

$$\Delta e(k^2) = -e\,\pi_{\gamma\gamma}/2k^2 = -e^3(f'_{DW} + f'_{DB})k^2/2\Lambda^2\,. \tag{41}$$

This is certainly the leading term of the universal electromagnetic coupling and for that reason is not worth to calculate the one-loop boson self-energies. Furthermore,

that and other oblique corrections have been annalized by several authors [4, 7, 8, 5] and will not be further studied here. Our primary interest were the non-universal flavour dependent electromagnetic form-factors. The result shown in eq. (39) contains no magnetic moment term and comprises a charge radius (CR) and one anapole moment (AM) whose values depend on the charge and isospin quantum numbers of each particle. But, since they are proportional to the electric charge both vanish in the neutrino case.

In processes at very low energies, the CR and AM contributions are not dynamically different from other local interactions such as the ones mediated by Z^0. That fact gives the opportunity for adopting different definitions of CR and AM. Although such a discussion is out of the scope of this work, we just add that within the presentation given in this section, the CR and AM arise as the local interaction couplings that survive in four fermion amplitudes if the source of the electromagnetic and Z^0 fields has a zero q_Z charge (eq. (29)). That is approximately true if the source (target) is a medium made of unpolarized electrons and/or protons. But in the work of Góngora and Stuart [19] for instance, the CR and AM are the couplings that survive if the source has a zero t_3 isospin component. One should keep in mind however, that what matters is the total amplitude which may comprise other local interaction contributions such as box diagrams, or even charged current interactions in the case of elastic scattering. Actually, both of them are produced by the operators $\mathcal{O}'_{DW}$, $\mathcal{O}_{DB}$ at one-loop level.

4 Overview and conclusions

Boudjema *et. al.* [11] calculated (in the unitary gauge) the fermion form-factors generated radiatively by the anomalous $WW\gamma$ interactions specified by the Lagrangian of eq. (14). We are now in position to recognize that the couplings they obtained are just the universal kind of vertices with only one exception. Keeping only the divergent terms, one has for a fermion with mass m and isospin $t_3 = \pm 1/2$:

$$\Delta\Gamma^{\mu}_{\gamma} = a\, \left(k^2\gamma^{\mu} - k^{\mu}\not{k}\right) T_3 - i\,\Delta\mu\,\sigma^{\mu\nu}k_{\nu}\,, \tag{42}$$

$$a = \beta\,\frac{\Delta k_{\gamma}}{M_W^2}\frac{\Lambda^2}{M_W^2} + 4\,\beta\,\frac{\lambda_{\gamma}}{M_W^2}\ln\frac{\Lambda^2}{M_W^2}\,, \tag{43}$$

where the anomalous magnetic moment takes the value (also calculated in refs. [9]):

$$\Delta\mu = 2\,m\,t_3\,\beta\,\frac{\Delta k_{\gamma}}{M_W^2}\ln\frac{\Lambda^2}{M_W^2}\,. \tag{44}$$

It is clear that the a term is a particular case of the universal interactions identified in eq. (28).

More recently, Hagiwara *et. al.* [8] calculated the one-loop fermion gauge couplings arising from the SU(2)xU(1) invariant operators $\mathcal{O}_{WWW}, \mathcal{O}_W, \mathcal{O}_B$. They restricted to the chiral conserving vector and axial-vector form-factors in the limit of zero fermion masses and found that $\mathcal{O}_B$ does not contribute and only $\mathcal{O}_{WWW}$ corrects the electromagnetic coupling. For a finite fermion mass the result is

$$\Delta\Gamma^{\mu}_{\gamma} = 6\,\beta\,\frac{f_{WWW}\,g^2}{\Lambda^2}\ln\frac{\Lambda^2}{M_W^2}\left(k^2\gamma^{\mu} - k^{\mu}\not{k}\right) T_3\,. \tag{45}$$

In view of the relations [8] between the parameters of the phenomenologic and effective Lagrangians (eqs. (1, 14)) namely,

$$\frac{\Delta k_{\gamma}}{M_W^2} = \frac{f_B + f_W}{2\Lambda^2}\,, \tag{46}$$

$$\frac{\lambda_{\gamma}}{M_W^2} = 3\,\frac{f_{WWW}}{2\Lambda^2}\,g^2\,, \tag{47}$$

there is agreement in the $\lambda_{\gamma} - f_{WWW}$ case (the $WW\gamma$ operators are exactly the same), but the two results seem in conflict in what regards the charge radius and anapole moment proportional to Δk_{γ}. That is not necessarily true because those are just the kind of universal couplings (eq. (28)) not expected to be independent

from the gauge-fixing condition. Boudjema *et. al.* worked in the unitary gauge and did not calculate the vacuum polarization functions whereas the 't-Hooft-Feynman gauge was used in ref. [8].

In conclusion, the non-universal electromagnetic form-factors contained in eqs. (42, 45) reduce just to a magnetic moment whose value is only significant (cf. eq. (44)), given the available experimental data, in the case of the muon. The associated $WW\gamma$ coupling proportional to Δk_γ is produced either in the operators $\mathcal{O}_W$ and $\mathcal{O}_B$ or in $\mathcal{O}_{BW}$ [4, 8]. In turn, the operator $\mathcal{O}_{WWW}$ only yields oblique corrections that simply renormalize the coefficients of $\mathcal{O}'_{DW}$ and $\mathcal{O}_{DB}$ [8]. The analysis of the low energy constraints on the oblique corrections performed by Hagiwara *et. al.* [8] and updated in [5] gives upper limits of the order of 2 and 40 to the absolute values of f'_{DW} and f'_{DB} respectively for a scale $\Lambda = 1\,\mathrm{TeV}$. The operators $\mathcal{O}'_{DW}, \mathcal{O}_{DB}$ do induce flavour dependent charge radius and anapole moments (eq. (39)) but not a magnetic moment. Therefore, although their quantum number structure is different from the tree-level oblique corrections, the magnitude is further suppressed by a factor β/e of the order of 10^{-4} and cannot be used to give new limits on the anomalous interactions. Finally, some anomalous $WW\gamma$ interactions also have effects on the flavour changing $b \to s\gamma$ transition [12, 13], but we checked that this is not the case of $\mathcal{O}'_{DW}$ and $\mathcal{O}_{DB}$.

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