# On the Use of 1-bit DACs in Massive MIMO Systems

João Guerreiro, Rui Dinis and Paulo Montezuma

Massive multiple input multiple output (MIMO) systems are being proposed for next generation broadband wireless systems. However, as the complexity of implementation grows with the number of antenna elements, their feasibility is a challenging task. In this work, we consider very low complexity, 1-bit digital-to-analogue converters (DACs) for the downlink of massive MIMO systems. We analyse the impact of the resultant severe nonlinear distortion effects when a low-complexity maximum ratio transmission (MRT) technique is employed for user separation. We show that the nonlinear distortion levels decrease with the number of transmit antennas allowing good performance, even with 1-bit DACs.

*Introduction:* It is widely known that multiple input multiple output (MIMO) techniques [1] allow significant performance improvements in wireless communication systems in terms of diversity and/or spatial multiplexing gains.

Recently, massive MIMO systems (i.e., MIMO systems with a very large number of antenna elements), combined with orthogonal frequency division multiplexing (OFDM) [2] techniques have been considered to meet the demanding requirements of fifth generation (5G) systems [3]. However, the implementation complexity of MIMO systems increases with the number of antennas, which makes the practical implementation of massive MIMO a challenging task [4].

In this work, we propose a very low complexity massive MIMO OFDM transmitter. Such low complexity is achieved with 1-bit digital-to-analog converters (DACs) at each one of its $T$ transmit antennas and a low-complexity maximum ratio transmission (MRT) technique for user separation, which has much lower signal processing requirements than singular value decomposition (SVD) or zero forcing (ZF) techniques. However, the use of 1-bit DACs leads to severe nonlinear distortion effects in the transmitted signals, which might preclude acceptable performance.

Taking advantage of the Gaussian-like nature of OFDM signals with a large number of subcarriers, we provide both simulated and theoretical results regarding the impact of severe quantization effects in the transmitted signals [5, 7]. More specifically, we provide the signal-to-interference (SIR) levels experienced at the detection stage, showing that for a given number of receive antennas (and assuming a different data stream for each one), the nonlinear distortion effects decrease with the number of transmit antennas, enabling the use of 1-bit DACs.

*System Characterization:* The massive MIMO OFDM system is composed by $T$ transmit antennas and $R$ single-antenna users. Each OFDM block is defined as $\mathbf{S}(r) = [S_{r,1}\ S_{r,2}\ \cdots\ S_{r,N_T}]$. Among the $N_T$ subcarriers, $N_U$ carry useful data symbols and $N_G = M(N_U - 1)$ are left idle to simulate an oversampling operation by a factor of $M = (N_U + N_G)/N_U$. The data to be transmitted is selected from a quadrature phase shift keying (QPSK) constellation, i.e., $S_{r,k} = \pm A \pm jA$ (the generalization for other cases is straightforward), where $\mathbb{E}[|S_{r,k}|^2] = 2A^2$. The time-domain OFDM signal transmitted to the $r$th user is the inverse discrete Fourier transform (IDFT) of $\mathbf{S}(r)$, i.e., $\mathbf{s}(r) = [s_{r,1}\ s_{r,2}\ \cdots\ s_{r,N_T}] = \text{IDFT}(\mathbf{S}(r))$. When $N_T$ is large, the time-domain samples $s_{r,n}$ have a complex Gaussian distribution with zero mean and variance $\sigma_s^2 = A^2/(N_U M^2)$. Thus, their real and imaginary parts have the following probability density function (PDF)

$$p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{s^2}{2\sigma_s^2}\right). \quad (1)$$

The channel between the $t$th transmit antenna and the $r$th user has $L$ taps and is represented by the vector $\mathbf{h}(r,t) = [h(1)_{r,t}\ h(2)_{r,t}\ \cdots\ h(L)_{r,t}]$, where $h(l)_{r,t}$ denotes the Rayleigh fading coefficient between the $r$th receive antenna and the $t$th transmit antenna for the $l$th tap. Employing OFDM, the frequency-selective channel associated to the massive MIMO system can be decomposed as a set of $N_T$ flat-fading channels. Therefore, for the $k$th subcarrier, we have

$$\mathbf{H}(k) = \begin{bmatrix} H(k)_{1,1} & H(k)_{1,2} & \cdots & H(k)_{1,T} \\ H(k)_{2,1} & \cdots & \cdots & H(k)_{2,T} \\ \vdots & \cdots & \ddots & \vdots \\ H(k)_{R,1} & \cdots & \cdots & H(k)_{R,T} \end{bmatrix}. \quad (2)$$

Thus, the received signal for the $k$th subcarrier is

$$\mathbf{Z}(k) = \mathbf{H}(k)\mathbf{S}(k). \quad (3)$$

*User Separation and Quantisation Effects:* Since we consider the MRT technique for user separation, there is pre-processing at the transmitter and the data symbols on the $k$th subcarrier are precoded by the transpose conjugate of (2), resulting

$$\mathbf{X}(k) = \mathbf{H}^H(k)\mathbf{S}(k). \quad (4)$$

The time-domain precoded OFDM signal of the $t$th branch is $\mathbf{x}(t) = [x_{t,1}\ x_{t,2}\ \cdots\ x_{t,N_T-1}] = \text{IDFT}(\mathbf{X}(t))$. The PDF of the samples $x_{t,n}$ is still approximately Gaussian, however, its variance is related to the number of users, i.e., $\sigma_x^2 = R\sigma_s^2$. Fig. 1 shows the PDF of the real part of samples $x_{t,n}$ considering OFDM signals with $N = 512$, $M = 4$ and a frequency-selective channel with $L = 128$ taps. The MIMO system has $T = 32$ transmit antennas and up to $R = 32$ single-antenna users. For

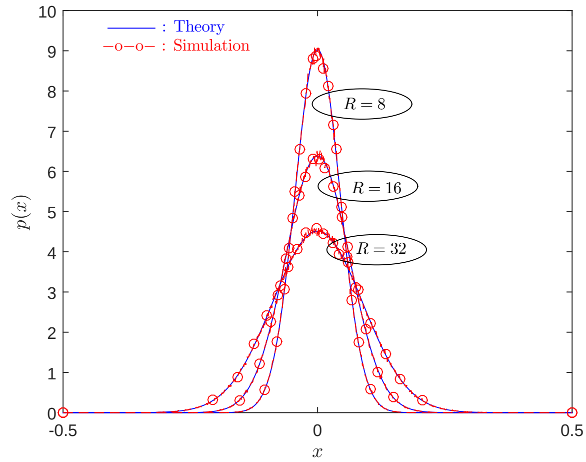


**Fig. 1** *PDF of the real part of a precoded OFDM signal using the MRT technique and a massive MIMO OFDM system with $T = 32$ and different values of $R$.*

each transmit branch we employ a 1-bit quantizer, that is characterized by the nonlinear function $f(x) = \text{sgn}(x)$, where $\text{sgn}(x)$ denotes the sign function. At the quantizer output, the signal to be transmitted is $\mathbf{y}(t) = [y_{t,1}\ y_{t,2}\ \cdots\ y_{t,N_T}]$, where

$$y_{t,n} = f(\text{Re}(x_{t,n})) + jf(\text{Im}(x_{t,n})). \quad (5)$$

However, due to the Gaussian distribution of the samples $x_{t,n}$, the Bussgang's theorem [5] holds and the quantizer output can also be expressed as the sum of two uncorrelated terms, i.e.,

$$y_{t,n} = \alpha x_{t,n} + d_{t,n}, \quad (6)$$

where

$$\alpha = \frac{\mathbb{E}\left[x_{t,n} y_{t,n}^*\right]}{\mathbb{E}\left[|x_{t,n}|^2\right]} = \sqrt{\frac{2}{\pi}}, \quad (7)$$

i.e., $\alpha$ is equal in all transmit branches, and $d_{t,n}$ represents the distortion term associated to the $n$th time-domain sample of the signal transmitted on the $t$th antenna. Additionally, it can be showed that

$$\mathbb{E}\left[d_{t,n} d_{t',n}^*\right] = 0, \quad (8)$$

for $t \neq t'$. Regarding the frequency-domain, and focusing on the $k$th subcarrier, we have

$$\mathbf{Y}(k) = \alpha\mathbf{X}(k) + \mathbf{D}(k), \quad (9)$$

with $\mathbf{D}(k)=[D_{1,k}\ \ D_{2,k}\ \ \cdots\ \ D_{T,k}]^T$ representing the nonlinear distortion terms. At the receiver side, we may write

$$\begin{aligned}\mathbf{Z}(k)&=\mathbf{H}(k)\mathbf{Y}(k)=\mathbf{H}(k)(\alpha\mathbf{X}(k)+\mathbf{D}(k)) \qquad (10)\\ &=\alpha\underbrace{\mathbf{H}(k)\mathbf{H}^H(k)}_{\mathbf{P}(k)}\mathbf{S}(k)+\underbrace{\mathbf{H}(k)\mathbf{D}(k)}_{\mathbf{W}(k)}.\end{aligned}$$

Moreover, from (10) the data received by the $r$th user on the $k$th subcarrier can be decomposed by three terms:

$$\begin{aligned}Z_{r,k}&=\alpha\sum_{j=1}^{R}P(k)_{r,j}S_{j,k}+W_{r,k} \qquad (11)\\ &=\alpha\underbrace{P(k)_{r,k}S_{r,k}}_{\text{Useful data}}+\underbrace{\alpha\sum_{j=1,j\neq k}^{R}P(k)_{r,j}S_{j,k}}_{\text{Inter-user interference}}+\underbrace{W_{r,k}}_{\text{Nonlinear distortion}},\end{aligned}$$

where $W_{r,k}$ denotes the weighted version of the nonlinear distortion terms

$$W_{r,k}=\sum_{j=1}^{T}H(k)_{r,j}D_{j,k}. \qquad (12)$$

The "overall noise" components of (11) are related with two things: (i) weaknesses of the MRT technique since, in general, $\mathbf{P}(k)$ is not diagonal, which is a necessary condition to avoid inter-user interference; (ii) the severe nonlinear distortion effects associated to the use of 1-bit quantizers in each transmit branch. However, in massive MIMO systems where $T\gg R$ and provided that the links between the transmit and receive antennas are uncorrelated, we have $\mathbf{P}(k)\approx T\mathbf{I}$ [6]. In these conditions, the data received by the $r$th user on the subcarrier $k$ is approximately

$$Z_{r,k}\approx\alpha T S_{r,k}+W_{r,k}, \qquad (13)$$

and the SIR is

$$\mathrm{SIR}_{k,r}^{\mathrm{MIMO}}\approx\frac{|\alpha|^2T^2\mathbb{E}[|S_{r,k}|^2]}{\mathbb{E}\left[|W_{r,k}|^2\right]}. \qquad (14)$$

However, as the distortion terms are uncorrelated at the transmission level (see (8)), it can be shown that $\mathbb{E}\left[|W_{r,k}|^2\right]=T\mathbb{E}\left[|D_{r,k}|^2\right]$. Therefore,

$$\mathrm{SIR}_{k,r}^{\mathrm{MIMO}}\approx\frac{|\alpha|^2T\mathbb{E}[|S_{r,k}|^2]}{\mathbb{E}\left[|D_{r,k}|^2\right]}. \qquad (15)$$

In a single-antenna nonlinear OFDM system the received signal is $Y_k=\alpha S_k+\tilde{D}_k$ and the SIR is [8]

$$\mathrm{SIR}_k=\frac{|\alpha|^2\mathbb{E}[|S_k|^2]}{\mathbb{E}\left[|\tilde{D}_k|^2\right]}. \qquad (16)$$

The comparison between the SIR associated to single input single output (SISO) and massive MIMO systems with MRT technique must take into account the power modification of the precoded signal. Thus, as the power of the precoded signal increases with $R$, we have

$$\mathrm{SIR}_{k,r}^{\mathrm{MIMO}}\approx\frac{|\alpha|^2T\mathbb{E}[|S_{r,k}|^2]}{R\mathbb{E}\left[|\tilde{D}_{r,k}|^2\right]}, \qquad (17)$$

and the relation between (16) and (17) is

$$\frac{\mathrm{SIR}_{k,r}^{\mathrm{MIMO}}}{\mathrm{SIR}_{k,r}}=\frac{T}{R}. \qquad (18)$$

This means that when $T\gg R$, the SIR is improved by a factor of $T/R$ when compared to the SIR associated to single-antenna OFDM systems.

*Performance Results:* Fig. 2 shows the simulated and theoretical SIR for a given user (the same results are observed for other users). The theoretical SIR was obtained by considering the approach of [7]. The simulation runs with OFDM signals with $N=256$ and $M=4$. We present the SIR for massive MIMO systems with $R=8$ users and different values of $T$. From the figure it can be noted that the SIR increases when $T$ increases. In fact, although the SIR does not increase by a factor of $T/R$ for all values of $T$, this is clearly verified for larger values of $T$ (for instance when $T$ increases from $T=32$ to $T=64$ or from $T=64$ to $T=128$), where the interference amongst users can be almost neglected. This means that even when 1-bit highly nonlinear

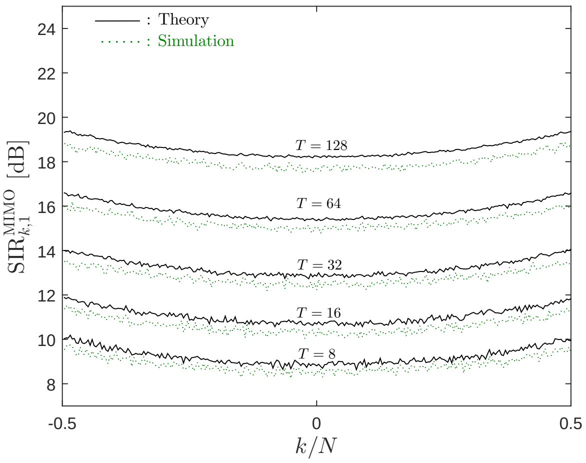


**Fig. 2** *Evolution of the SIR for a given user considering $R=8$ and different values of $T$.*

quantizers are employed, the corresponding nonlinear distortion effects can be reduced provided that $T\gg R$.

*Conclusions:* In this paper we analyse the impact of quantization effects in massive MIMO systems that employ an MRT technique combined with 1-bit DACs. It is shown that it is possible to have relatively low distortion levels for detection purposes, provided that the number of transmit antennas is much higher than the number of different data streams.

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João Guerreiro and Rui Dinis (*Instituto de Telecomunicações, FCT - Universidade Nova de Lisboa, Lisboa, Portugal*)

Paulo Montezuma (*UNINOVA, FCT - Universidade Nova de Lisboa, Lisboa, Portugal*)

E-mail: jf.guerreiro@campus.fct.unl.pt

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